A HOLISTIC ASSESSMENT OF SLOPE STABILITY ANALYSIS IN MINING APPLICATIONS

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1.0 INTRODUCTION

Slope stability analysis plays an integral role in the design of various mining applications including waste dumps, heap leach piles, solution ponds, and tailings dams. Generally, limit equilibrium analysis using one of the several prevalent approaches is considered adequate. The density, saturation, and shear strength parameters of the materials forming the slope affect the failure mode and the calculated factor of safety (FS) against sliding. These parameters are generally based on laboratory tests. Field practices and construction procedures are often not completely simulated in the laboratory for various reasons (e.g. equipment limits, time and budget restraints, etc.).

This paper presents a holistic assessment of slope stability analysis as practiced in mining applications, using example data from multiple heap leach projects. A sensitivity analysis is presented for variations in material properties, data interpretation, and computation methods. For each step in the design process, the possible variations in parameter values were identified and then used to perform traditional and probabilistic stability analyses. This simple, cradle-to-grave-type approach is used to evaluate the reliability of an example design, and the combined impact of multiple uncertainties on the factor of safety.

1.1 Example Study

The issue of addressing uncertainty in geotechnical design has been discussed in depth by numerous authors (Duncan 2000; Christian 2004; Whitman 1984; Christian et al. 1993). One may ignore the uncertainties involved in a design, take a conservative approach, rely on observational methods (Peck 1969), or attempt to quantify the uncertainty. Geotechnical projects in general, may include a combination of these methods.

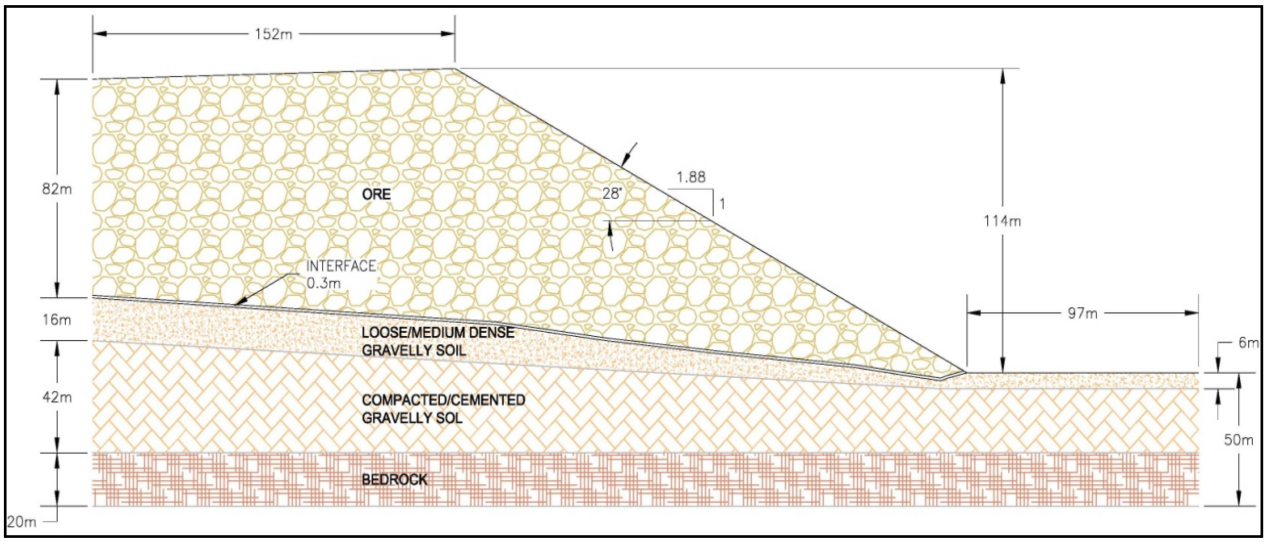
For important structures, such as heap leach pads, it is critical that sources of uncertainty in the stability analysis be acknowledged early on and considered in the overall design approach. As with any project, economics and other physical constraints, such as space limitation, often do not always allow for an overly-conservative, robust design. In an effort to quantify uncertainty and provide a sense of level of confidence in the safety and reliability of a design, probabilistic methods have been developed and implemented in many slope stability software packages. Reliability methods are often used in the design of open pit mine slopes, but not as commonly in designing heap leach pads and waste dumps.

As an example, the stability analysis of a copper heap leach project is presented here to evaluate the effects of multiple sources of uncertainty and differing methods of data interpretation. Some of the parametric values, or the variation therein, are assumed on the basis of actual data from multiple heap leach projects, included in the paper as well. A generic representation of the example case study is shown in Figure 1. As depicted in the cross-section, the ultimate height of the design is 114 m (measured from the crest to the toe). The overall slope of the heap leach pad is 1.88 horizontal to 1 vertical (1.88H:1V), or 28°. The slope benches are considered in the overall slope.

The example leach pad is founded on alluvial, colluvial and residual soils overlying weathered limestone. The ore to be placed on the pad is characterized as poorly graded gravel (GP) with average fines content (percent passing #200 sieve) of 4%. The liner subgrade is low permeability (fine) soil. The cover or the drainage material, placed directly above the geomembrane (between the liner and the ore), is crushed ore in this case. The phreatic surface was assumed to be 1 m above the base liner, which is what the collection system over the liner is typically designed for.

In heap leach pads, typically, Linear Low Density Polyethylene (LLDPE) or High Density Polyethylene (HDPE) is used as the base liner. The decision is based on the elongation, strength and other requirements of the application as well as economic considerations. In this example study, the base liner was 80-mil single-side textured LLDPE.

Figure 1: Cross-Section for Example Study



2.0 FIELD INVETIGATION AND SAMPLING

When selecting appropriate values for the input parameters of the stability analysis, the level of uncertainty in the data and the assumptions that are made must be clearly identified and considered in the design. This concept has been emphasized through an extensive number of publications regarding geotechnical uncertainty and reliability (Christian et al. 1994; Duncan 2000; Christian 2004). The primary source of uncertainties involved in slope stability analysis for mining applications is inadequate geotechnical investigation, often lacking in a thorough assessment of in-situ material characterization and sampling disturbances. To emphasize this point, some background information is presented here.

The tradeoff between the costs of a thorough site investigation versus the risks of design uncertainty has long been a challenging management decision in geotechnical projects. For mine sites, significant investment is typically made in exploration and estimating mineral resources and the geology of a mine site is often more thoroughly documented than other types of geotechnical projects. Nevertheless, the engineering properties of the soil and rocks relevant to slope stability receive less emphasis. Baecher and Christian (2003) observed that the areas of geotechnical concern, such as slopes and waste disposal facilities, are usually associated with mine costs rather than revenue, and therefore, significantly less money is devoted to their site characterization and laboratory testing.

The expenditure for site investigations varies significantly from project to project, with higher levels of uncertainty and, therefore, the potential for increased construction costs, typically associated with lower level investigations (Whyte, 1995). In 1984, the National Research Council recommended that a minimum of 3% of total project cost be dedicated to site investigation to reduce the level of uncertainty in civil design projects. Considering the high costs of mining projects, the percentage of the total project cost spent on geotechnical investigation is in reality much less. In heap leach projects, the properties of the materials directly involved with the leaching process (e.g. ore, overliner, geomembrane liner, etc.) usually are the focus of the laboratory testing budget, with foundation materials related to global stability receiving far less attention. For the example study presented in this paper, the foundation of the proposed leach pad area consists of a 16-m thick, loose to medium dense gravelly soil layer underlain by a thicker dense gravel layer and bedrock (Figure 1). By way of geotechnical investigation for the foundation, a total of two samples of the upper layer of gravelly soil were collected. Trying to estimate geotechnical properties based on a very limited number of data points is a common issue due to limited site investigations, and can result in significant bias in the analysis (Christian 2004). Some heap leach projects are initially designed without any laboratory testing of the existing foundation material, and typical properties for that material type are assumed in the analysis, introducing additional uncertainty.

For the example study, the two samples obtained were subjected to typical disturbance effects. The sampling procedure, transportation to the laboratory, and preparation for testing change the original stress condition of the soil and alter the soil structure to one extent or the other, and therefore affect the behavior of the material (Lambe and Whitman 1969). The potential effects of sample disturbance on laboratory results have been widely researched and discussed for various soil types for decades (Schmertmann 1955; Ladd and Lambe 1963; Lambe and Whitman 1969), but the effects on the results of the stability analysis are often not considered thoroughly during selection of the input parameters.

For fine-grained soils, disturbance typically results in decreased undrained strength and increased compressibility. Sample disturbance effects for gravelly soils have also been widely investigated for seismic stability and liquefaction concerns. For cohesionless soils like silty-sand and gravels, significant effort may be required to avoid disturbance of the soil fabric. However, Mori and Koreeda (1979) concluded that “the angle of internal friction obtained from consolidated drained tests is not sensitively influenced by the disturbance of samples”, and attributed this phenomenon to factors unaffected by sample disturbance (e.g., grain size distribution and fines content) having the greatest influence on strength parameters obtained for cohesionless soils. On the other hand, Zlatovic and Ishihara (1997) found that the residual (post-peak) strengths are significantly affected by the sample preparation methods inducing varying levels of change in the specimen soil fabric.

3.0 LABORATORY TESTS - VARIABILITY OF MEASURED PROPERTIES

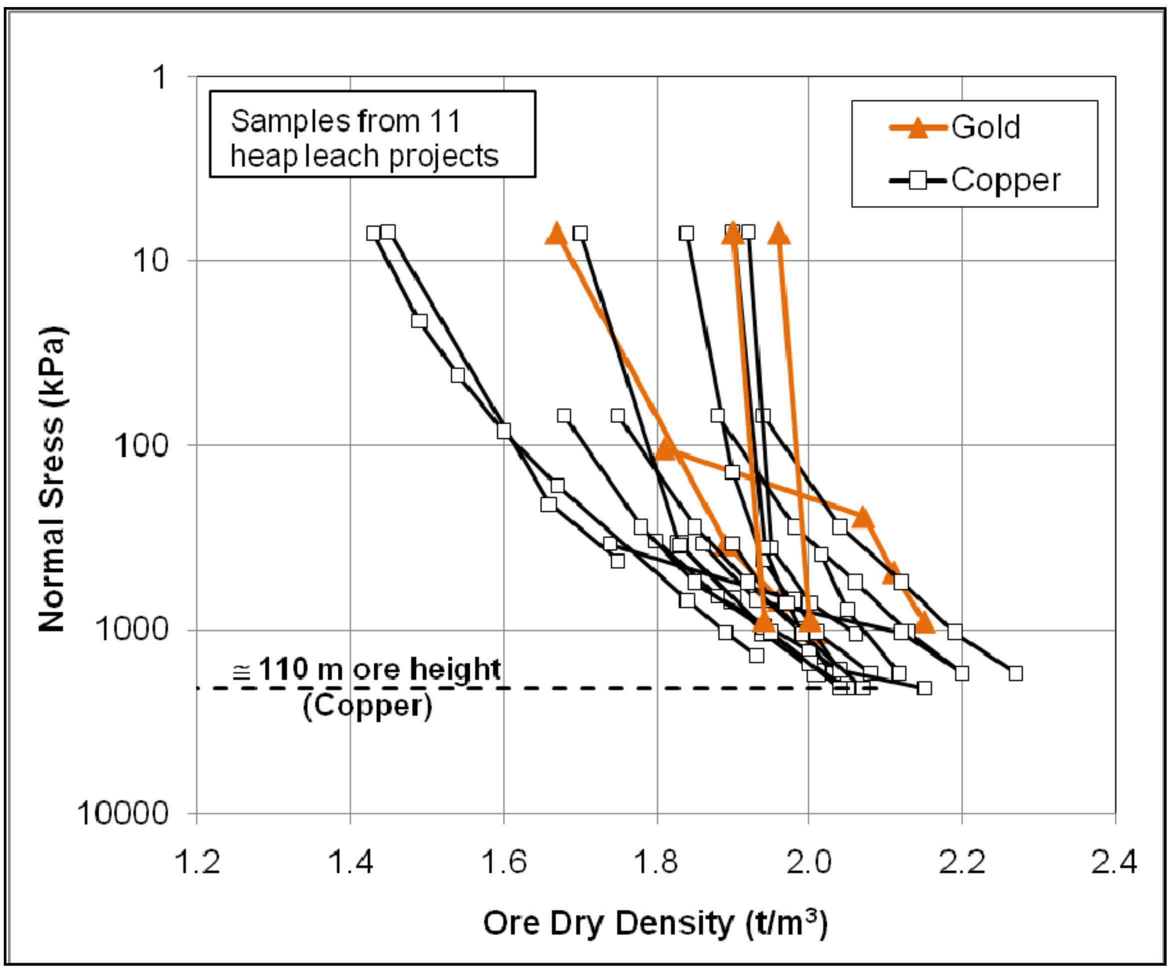
Just as with any other analysis, the results for a slope stability analysis are only as reliable as the input data. Beyond the sample disturbance and representation of in-situ variability, the uncertainty in input data is governed by the number and accuracy of laboratory tests conducted to determine the material properties. For most of the parameters for the example presented in this paper, a limited number of tests were performed and there is a high degree of uncertainty regarding variability in the parametric values. Ideally, considerable amount of sampling and testing must be performed so that the natural variability of the geotechnical material and relevant properties are adequately represented and the expected range of values is well defined. Unfortunately, as described previously, this is not the typical case.

From the slope stability analysis point of view, heap leach facilities present some unique problems and challenges. When heap leach pads were first utilized, the typical maximum ore depth was around 15 m (Thiel and Smith, 2003). Currently, much larger leach pads are utilized with design depths exceeding 200 m inducing considerably variable conditions within the heap. Some of these conditions and their implications are discussed below and are subsequently used as the basis for selecting input parameters and the range of their variations for the example case study.

3.1 Ore Density

Stacking or dumping fresh ore in 4-m to 6-m thick lifts (as commonly used in gold and copper heap leach pads) can result in significant variability of the in-place density within the heap. There are density variations within the heap due to change in normal load with depth also - as the depth within the heap increases, the ore density increases due to higher normal stress and greater compaction. The ore density may also increase and vary due to equipment traffic movement and induced compaction. Dumped ore densities in heap leach pads typically range from 1,500 kg/m^3 to1,800 kg/m^3 (Thiel and Smith, 2003).

Figure 2. Variations in Ore Dry Density



In addition to typical in-situ density variations induced during stacking and operation, continued chemical reactions in leached ore samples may produce variation in laboratory test results, depending on time elapsed since active leaching was stopped. These issues have been particularly evident in some recent nickel projects, where the moisture content of the compaction test specimens was difficult to control and maintain and the maximum proctor densities varied from 1.27 t/m^3 to 1.70 t/m^3, and the optimum moisture contents ranged from 29 % to 38%.

For the case example investigated here, the maximum dry density and optimum moisture content obtained from a single set of Proctor compaction tests were 1.9 t/m^3 and 10 %, respectively. Figure 2 shows the variation in ore dry densities for gold and copper samples obtained from previous testing for a number of heap leach projects. No effort is made to separate the causes of variations in presenting this data. An increase in the unit weight of the ore could increase the driving as well as resisting forces on the failing mass, depending on the type of slope failure. Thus, the data raises the question if such density variation through the vertical profile of the heap is significant enough to affect the stability results, and should this be represented in the modeling effort? In order to address this issue of variation of density, a range of dry densities (1.7 t/m^3 to 2.2 t/m^3) was assumed in the stability analysis for the example case. This range encompasses the value of 1.9 t/m^3 obtained in the single laboratory test, and at 10% moisture content corresponds to bulk unit weights of 16 kN/m^3 to 24 kN/m^3.

3.2 Shear Strength

3.2.1 Nonlinear Strength Envelopes

Ore density and gradation variability along with differences in normal and confining stress (e.g., inside the pile versus at the toe or on the slope face), result in heterogeneous shear strength throughout the pile. Generally, a linear strength envelope with a single friction angle value over the entire range of stresses is assumed for stability analysis. However, pile heights achieved these days result in a much wider range of normal stress variation in the pile, over which the strength envelope does not necessarily remain linear.

Figure 3. Variations in Ore Strength

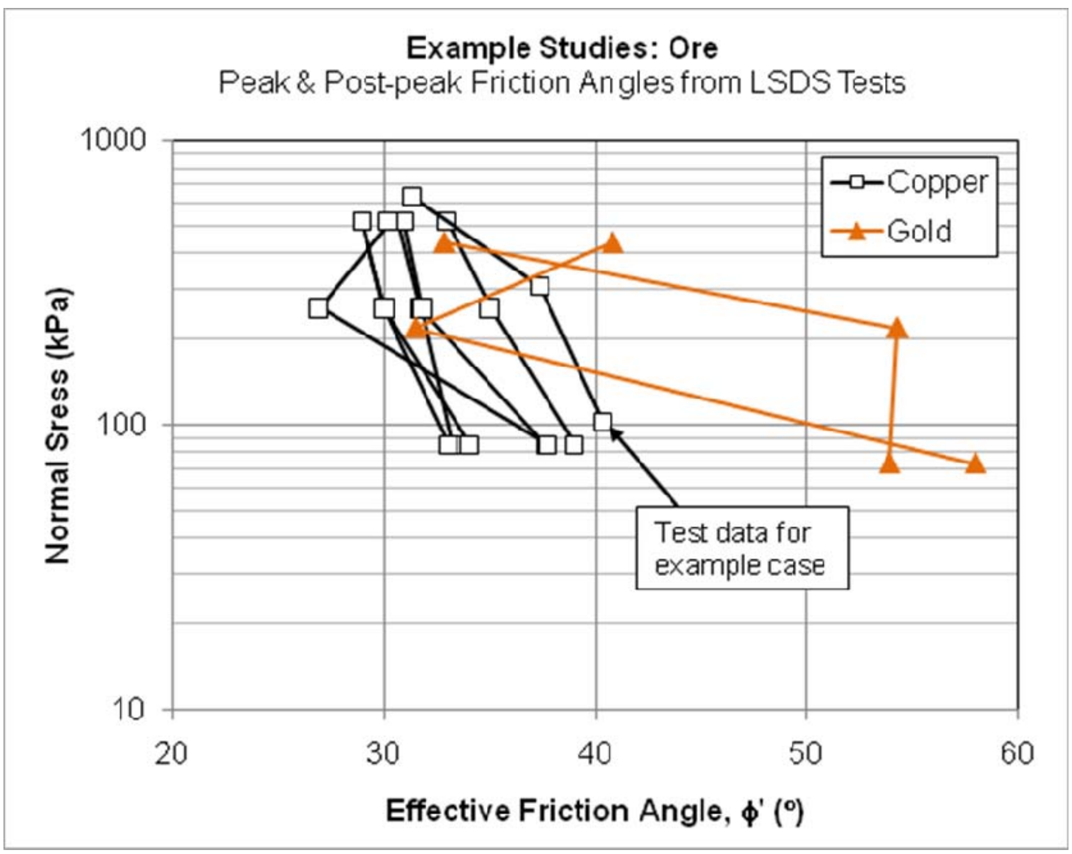


Figure 3 shows some results obtained from large scale direct shear (LSDS) tests on ore samples from various types of heap leach projects. The failure envelope in each case was nonlinear and the figure shows effective friction angles obtained over three separate stress ranges for each ore type, plotted against the mid-point of the respective stress range. The curvature of the shear strength envelope results in decreasing friction angle with increasing normal stress for all of the samples. The significance of the effect of the curvature of the strength envelope must be evaluated on a case-by-case basis, however, and the loading conditions in the specific application must be considered. In some cases, if high stress conditions do not exist, linear interpolation with lower cohesion value may result in more conservative design than use of the nonlinear envelop.

For the example case study, a single ore specimen was used to evaluate the internal strength of the ore. The effective area of the test specimen was 0.3 m x 0.3 m (12 in x 12 in) for the LSDS test. The ore was sheared at normal stresses of 207 kPa (30 psi), 414 kPa (60 psi), and 862 kPa (125 psi). These stresses correspond to approximately 10 m, 20 m, and 43 m meters of ore height for our case study. The LSDS test data for the ore is shown in Figures 4 and 5. The least square best-fit linear approximation of the strength envelope gives the effective friction angle for the ore as 37° and the cohesion as 15 kPa. Using a nonlinear envelope fit to the same data, the effective friction angle can be defined as 40°, 37°, and 31° for the stress ranges 0 kPa to 207 kPa, 207 kPa to 414 kPa, and 414 kPa to 862 kPa, respectively. The cohesion ranges from 0 kPa (value assumed at zero normal stress for the curved envelope) to 80 kPa for the same stress ranges.

The liner interfaces with the overliner (the drainage material), the subgrade, or the ore material itself (in case of interlift liners) create planes of weakness in the leach pile. Slides in lined facilities usually occur by wedge failure along the geomembrane interface with geotextile or low-permeability subgrade (Breitenbach 1998), this being the weakest link of the chain. Thus, the soil/liner interface strength parameters become the most critical data for evaluation of heap leach stability. The soil/liner interface strength depends on several factors, including normal load, rate of applied shear, soil type, density, and water content, liner thickness, flexibility and texture, and drainage condition.

Figure 4. Example Case – LSDS Test Data for Ore

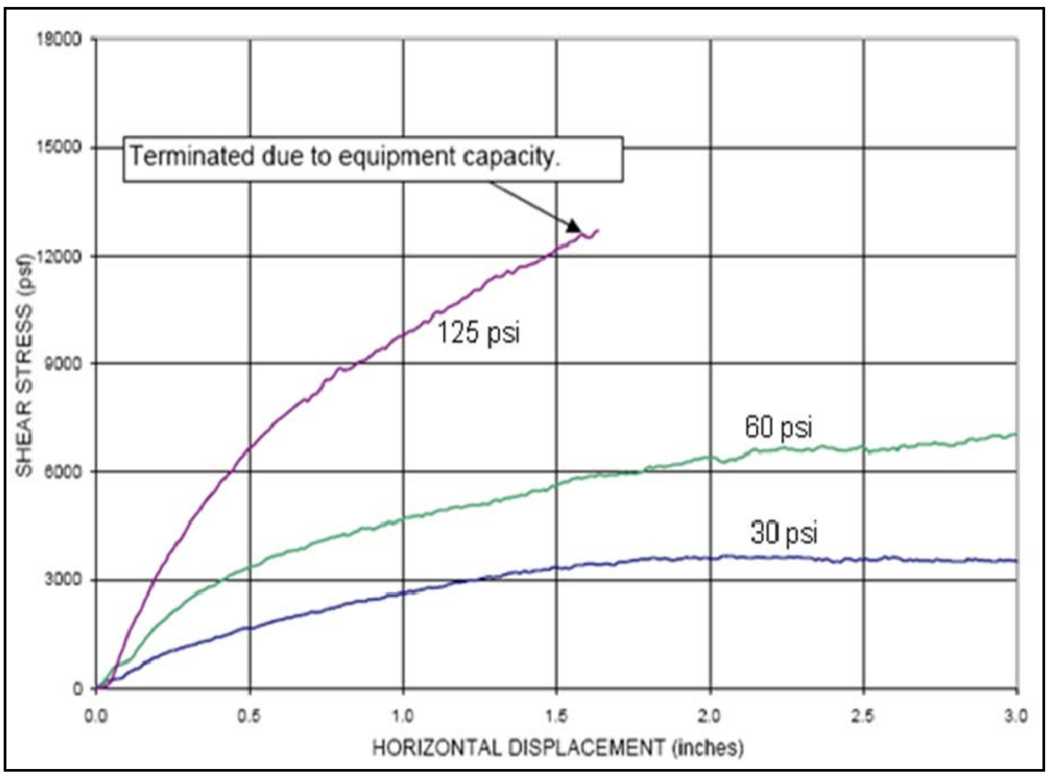
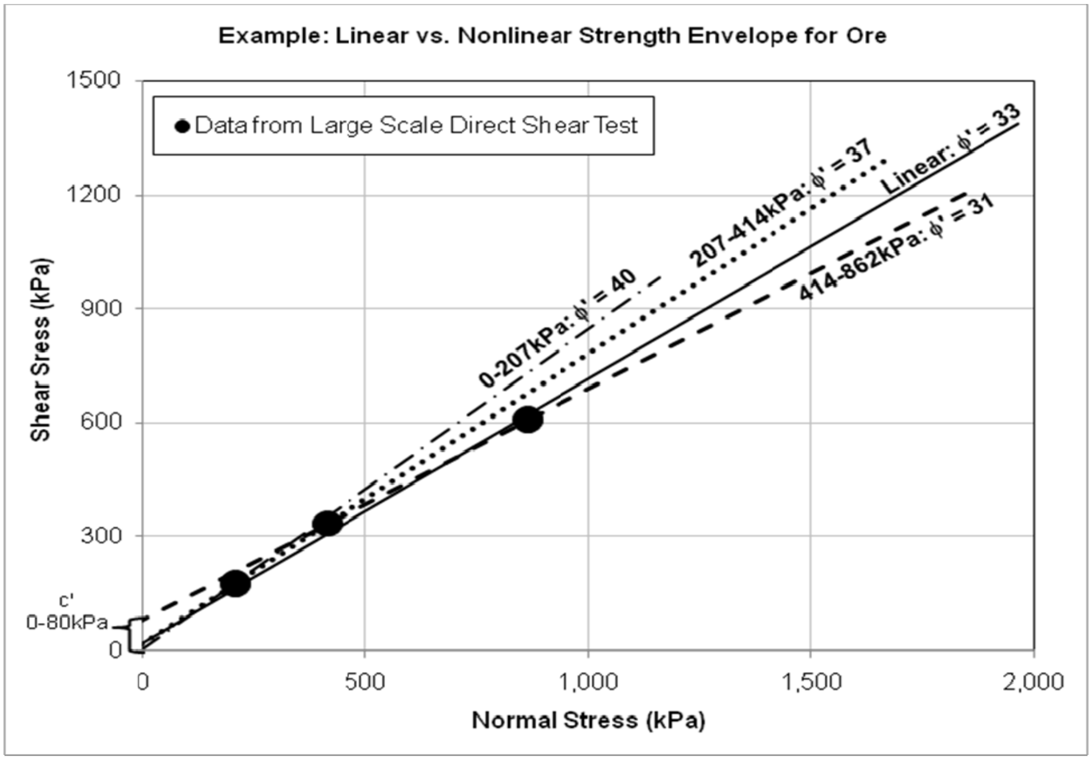


Figure 5. Example Case – Strength Envelope for Ore



Just as the ore material itself, soil/liner interface strengths may also exhibit a nonlinear strength envelope, with the friction angle generally decreasing as the normal stress increases. Data from LSDS tests on ore/liner and subgrade/liner interfaces from several projects with similar materials to the example case are presented in Figures 6 and 7. The plots shown for both of the interfaces indicate significant decreases in friction angle with increasing normal stress. Thus, as heap leach piles are extended to greater heights, decreases in the interface friction angle used for the stability analysis should be considered for the liner interface, as well as the ore.

Figure 6. Ore/LLDPE Interface Strengths for Copper Projects

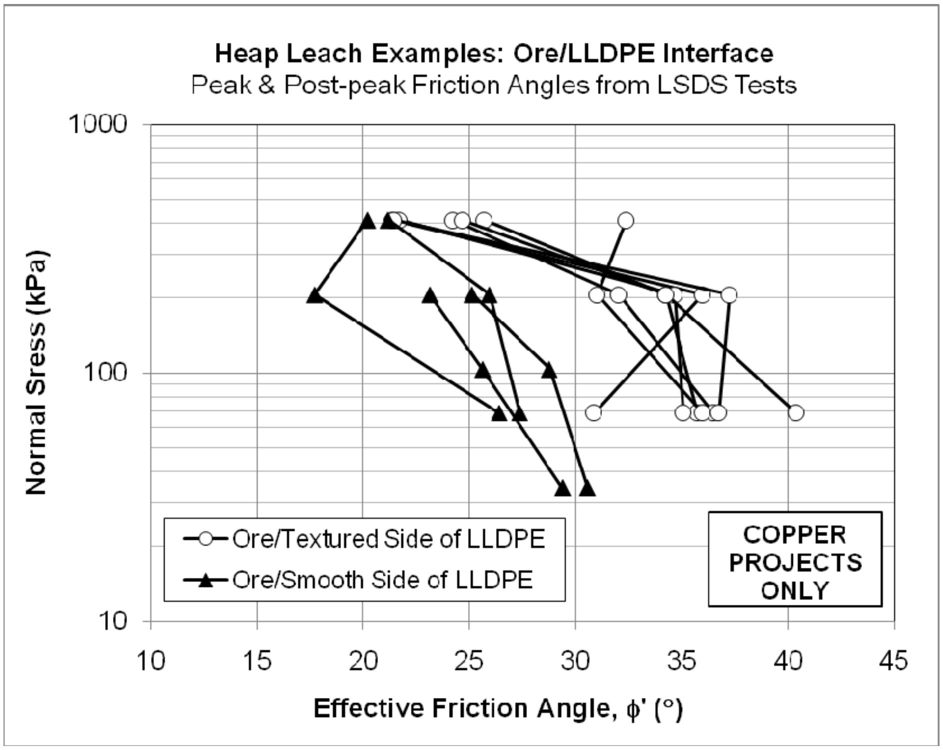
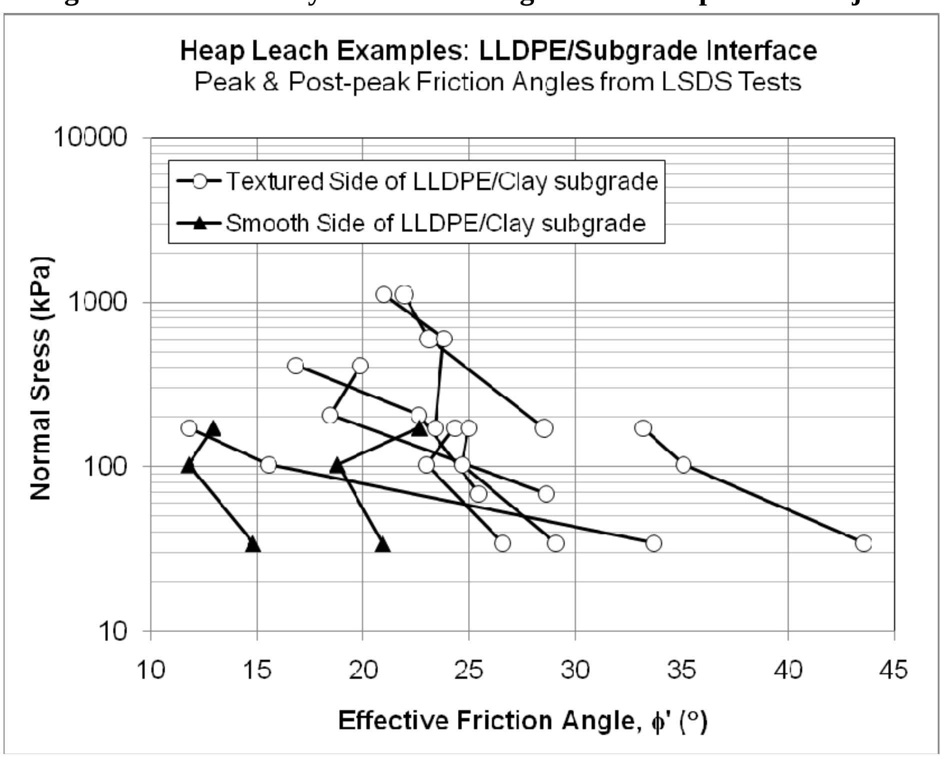


Figure 7. LLDPE/Clay Interface Strengths from Heap Leach Projects



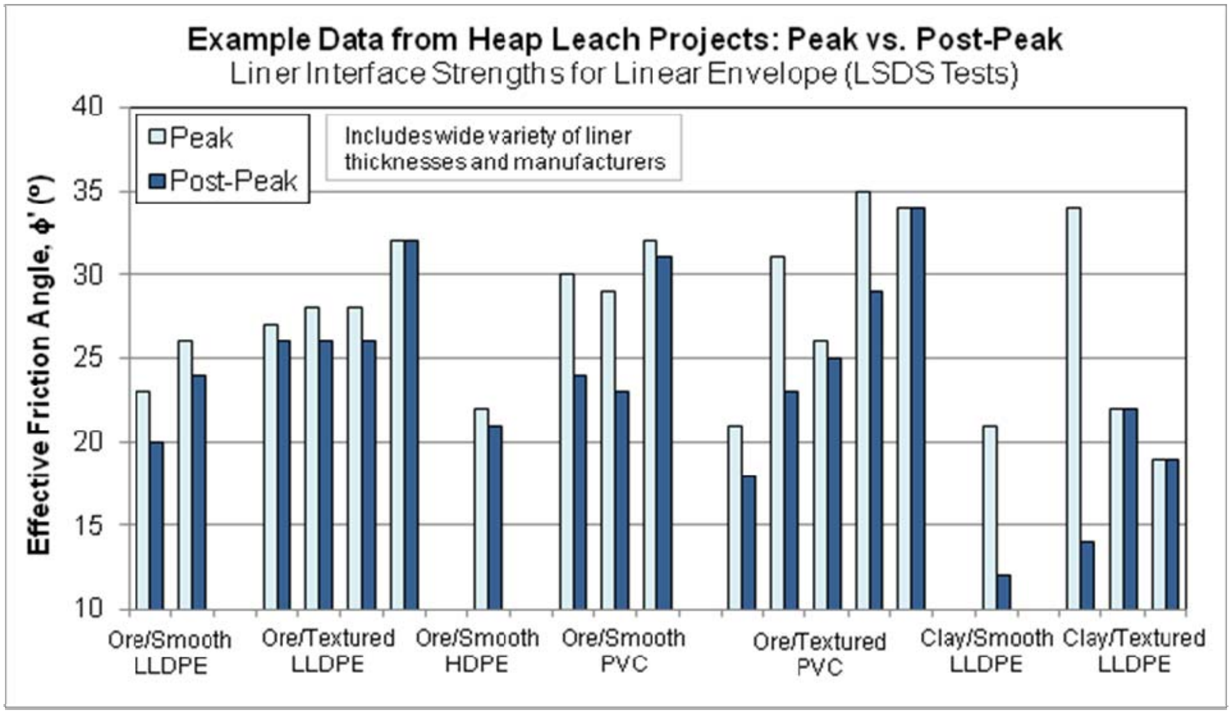
For the example study, two LSDS tests were performed to determine strength parameters for the LLDPE liner/clay subgrade and the ore (used as overliner material)/liner interfaces. As expected and stated previously, the LLDPE/clay subgrade interface was more critical (i.e. lower strength) than the ore/liner interface. A linear strength envelope fitted to the LSDS test data for the LLDPE/clay subgrade results in an effective friction angle of 22° and cohesion of 46 kPa. A nonlinear envelope gives friction angles of 35°, 27°, and 20° for the stress ranges 0 kPa to 103 kPa (0 kPa to 15 psi), 103 kPa to 207 kPa (15 psi to 30 psi), and 207 kPa to 414 kPa (30 psi to 60 psi), respectively. The cohesion ranges from 0 kPa (value assumed at zero normal stress for the curved envelope) to over 50 kPa for the same stress ranges.

No such variation was considered for the gravelly foundation material and an effective friction angle of 38° and zero cohesion, based on a linear approximation of the strength envelope, were used in the analysis.

3.2.2 Peak and Residual Strengths

To ensure conservative design, post-peak (residual) strengths are generally used for slope stability analyses. Numerous studies of shear stresses for geomembrane/soil interfaces based on direct shear testing have been published, and the conclusions regarding peak versus post-peak strengths have been mixed. Post-peak strengths as low as 50 % of peak strength have been observed for geomembrane/clay interfaces (Byrne 1994; Stark 1994), while other studies indicated that no strain-softening behavior occurred (Koerner et al. 1986, Masada et al. 1994). Valera and Ulrich (2000) recommend the use of post-peak shear strength for soil/liner interfaces in stability analyses of heap leach pads, because the interface may reach residual strengths due to minor strains caused by installation and initial loading. Sharma et al. (1997) observed that the reduction in HDPE/soil interface strength after peak stress was greater when the plasticity index of the soil was more than 30. The example data presented in Figures 6 and 7 is re-plotted below in Figure 8 to illustrate potential differences between the peak and residual strengths.

Figure 8. Peak and Post-Peak Liner Interface Strengths for Some Copper Projects



No strain softening behavior was observed for the ore or the liner/subgrade interface tested for the example case study and the friction angles used for the analyses were based on the peak strengths shown in Table 1 and discussed previously.

4.0 STABILITY ANALYSES AND INPUT VALUES

4.1 General

Global stability analyses for the case example were performed using a commercial software (Rocscience) based on the limit equilibrium approach. Circular failure surfaces through the ore, as well as block failure with sliding along the liner interface were analyzed for static loading conditions. The software internally divides the failing mass into vertical sections ("slices") and considers the equilibrium of each slice using a number of methods of analysis built into the software. The critical failure surface associated with the minimum factor of safety is located through a number of search methods, and the factor of safety is calculated.

For the example case study, factors of safety were calculated using the methods commonly identified as "Bishop simplified," "Janbu corrected" and "Spencer". The three methods yielded very similar results regarding the relative sensitivity of the calculated factor of safety to each input parameter. Therefore, for simplicity, only the results from the Spencer method were chosen for presentation here.

To examine the sensitivity of the analysis to a particular parameter, all other parameters were held constant while the parameter of interest was varied through the range of values defined in earlier sections. The base values for each of the parameters, which are referred to as the Most Likely Values (MLV), are shown in Table 1. The maximum and minimum values used for each parameter are referred to as the Highest Conceivable Value (HCV) and Lowest Conceivable Value (LCV), respectively. The LCV to HCV range represents the span expected to encompass approximately 3 standard deviations from the mean (including 99.7% of the "data"). The LCV and HCV values chosen for each parameter are shown in Table 2 and the bases for their selection are discussed subsequently.

Table 1: Most-Likely-Values (MLV) Assumed for Example Case

Material	Unit Weight	Effective Stress-Based Shear Strength Parameters	
		Cohesion, c′	Friction Angle, ϕ′
	(kN/m^3)	(kPa)	(°)
Ore	20.0	15	37
Interface	18.0	46	22
Loose/ medium dense Gravelly Soils	16.0	0	38
Compact/Cemented Soils	20.0	15	40
Bedrock	21.6	320	50

Table 2: Lowest and Highest Conceivable Values (LCV and HCV) Assumed for Example Case

Material	Input Parameter	LCV	MLV	HCV
Ore	Unit Weight	16 kN/m^3	20 kN/m^3	24 kN/m^3
	Friction Angle	30°	37°	40°
	Cohesion	0 kPa	15 kPa	25 kPa
Liner/Subgrade Interface	Friction Angle	17°	22°	27°
	Cohesion	0 kPa	46 kPa	60 kPa
Upper Foundation layer	Friction Angle	29°	38°	44°

In addition to the parameters shown in Table 1, the sensitivity of the results to the search method used for detecting the critical failure surface was investigated also. The minimum factor of safety (FS) results are typically more sensitive to adjustments in the critical failure surface searching method than the limit-equilibrium method chosen (e.g., Spencer, 1967; Bishop, 1955)

4.2 Sampling and Disturbance Effects for Foundation Soil

The potential sample disturbance effects and uncertainty due to spatial variability were evaluated for the foundation layer in our case study. Since the slope failure surfaces are not expected to penetrate deep into the lower foundation layer, only variations in the properties of the upper foundation layer (loose to medium dense gravelly soil) were evaluated. The assumed MLV of 38° for the effective friction angle for the upper foundation layer (loose to medium dense gravelly soil) was based on limited direct shear tests. Direct shear tests have been reported to introduce errors in measurement of friction angle as high as 2 degrees (Lambe and Whitman, 1969). Additionally, in loose to medium dense gravelly soil, 6° to 7° of difference in the friction angle may exist between the loose and dense gravel pockets. Based on these considerations, the LCV and HCV for the effective friction angle of the upper foundation layer have been assumed as 29° and 44°, respectively. These LCV and HCV values roughly correspond to a range encompassing 3 standard deviations from the mean, giving a coefficient of variability (standard deviation/mean) for the friction angle as 7%. Schultz (1972) reported a coefficient of variation of 7% for the peak friction angle of gravels, indicating that the assumed range for the case study is quite reasonable.

4.3 Ore Density Effects

The MLV for the unit weight of the ore based on the available test data is assumed as 20 kN/m^3. Based on a conservative evaluation of data from other copper heap leach projects, the HCV and LCV for the ore density were chosen as 24 kN/m^3 and 16 kN/m^3, respectively.

4.4 Strength Property Effects

<u>4.4.1 Ore</u>

As indicated in Table 1, the MLV of the effective friction angle and cohesion for the example case were chosen as 37° and 15 kPa, respectively. These values were obtained by fitting a linear strength envelope to LSDS test data. For geotechnical testing of all soil types, the typical range in coefficients of variation for measured effective friction angles has been reported as 2 % to 13 % (Duncan, 2000; Harr, 1984; Kulhway, 1992), while, several authors have reported coefficients of variation for cohesion ranging from 10 % to 40 % (Duncan 2000; Harr 1984; Fredlund and Dahlman 1972). Using these findings and the test data presented in Figure 3 as a guideline, the LCV to HCV range for the effective friction angle and cohesion of the ore in the example study were chosen as 30° to 40°, and 0 kPa to 25 kPa. These values provided coefficients of variations of 5% and 28%, respectively.

<u>4.4.2 Liner</u>

The MLV selected for the liner interface was based on the linear approximation of the test data for the most critical interface (i.e. the LLDPE/clay subgrade interface). The MLVs for the effective friction angle and cohesion of this interface were 22° and 46 kPa, respectively. The HCV and LCV for the effective friction angles were chosen as 17° and 27°, respectively. These values were based on the test data shown in Figures 6 and 7, and published observations about typical variations in strength properties. The liner interface strength is dependent upon multiple factors introducing additional sources of uncertainty, and therefore the range of values chosen represent a high coefficient of variation (22%). The standard deviation for the effective friction angle is approximately 1.7 degrees.

Measurements of cohesion for liner interfaces are less reliable than effective friction angle measurements, and often zero cohesion is assumed in stability analyses of lined facilities. Therefore, the LCV for the interface cohesion was assumed to be zero. When test data is available for the site specific materials, such as with the example case study, nonzero cohesion values may be used with caution. As mentioned before, for geotechnical tests on all types of materials, Duncan (2000) suggested a coefficient of variability as high as 10 % to 40 % for measured cohesion. In the present case, based on the linear strength envelope for the test data, the MLV for cohesion was 46 kPa. This value was higher than typically measured (i.e. 15 kPa to 30 kPa) for geomembrane/clay interface LSDS tests, based on an extensive database compiled by Vector's laboratory. Since the cohesion value was already greater than typical test results, the HCV was limited to 60 kPa.

5.0 STABILITY ANALYSES RESULTS

The results of the stability analyses showing the sensitivity to each of the parameters investigated are shown in the form of bar charts in Figures 9 and 10. These figures are self explanatory, but a brief description of the influence of each parameter investigated is given below.

Foundation Material: Since in the example case, the failure surface typically occurs in the weaker materials above the foundation layer, the variability in the strength of the foundation layer does not have a significant effect on the slope stability. Generally, this may be the case in other heap leach and waste dump applications unless the foundation layer is exceptionally week.

Ore Density: The ore density (and unit weight) variation within the range assumed, did show a slight decrease in factors of safety with increasing density, but not significantly. For a uniform ore density and unit weight profile, the factors of safety for failure through the liner for the LCV, MLV and HCV unit weight values (16, 20 and 24 kN/m^3) were 1.52, 1.48 and 1.45 respectively. Similar effects were observed for circular failure through the ore, as shown in Figure 10. The factors of safety for the non-uniform profile with unit weight ranging from 18 kN/m^3 to 24 kN/m^3 were not much different (within 1% of those obtained for the uniform profile based on the MCV of unit weight).

Ore Strength Parameters: Variation in the friction angle of the ore had a pronounced impact on the stability in both block failure along the liner and circular failure. For the LCV-HCV range of 30°-40°, the factors of safety varied from 1.25 to 1.57 for failure along the liner, and from 1.27 to 1.8 for circular failure. Cohesion of the ore material had a much less pronounced effect and the factors of safety varied from 1.43 to 1.5 for failure along the liner and 1.41 to 1.7 for circular failure for ore cohesion values ranging from 0 kPa to 25 kPa.

Ore Strength Envelope Shape: For the example case, using a nonlinear strength envelope to more accurately represent the test data for the ore, in comparison to a linear strength envelope over the entire stress range, resulted in only slightly lower factor of safety. For example, for the MLV case with failure occurring through the liner, the minimum FS is 1.45 and 1.48 for the linear and nonlinear strength envelopes, respectively, using the Spencer method. A similar, almost negligible difference was observed for the circular failure mode through the ore, as indicated in Figure 10.

Figure 9. Sensitivity of Analysis for Failure Along Liner Interface

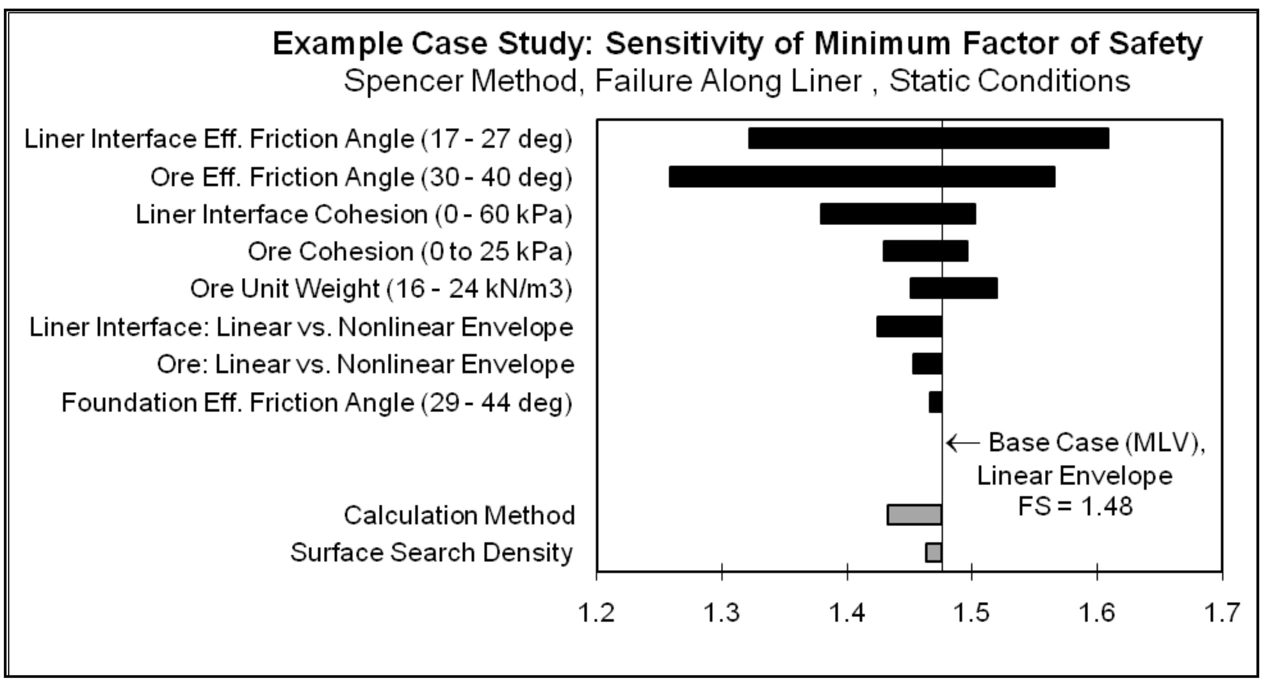
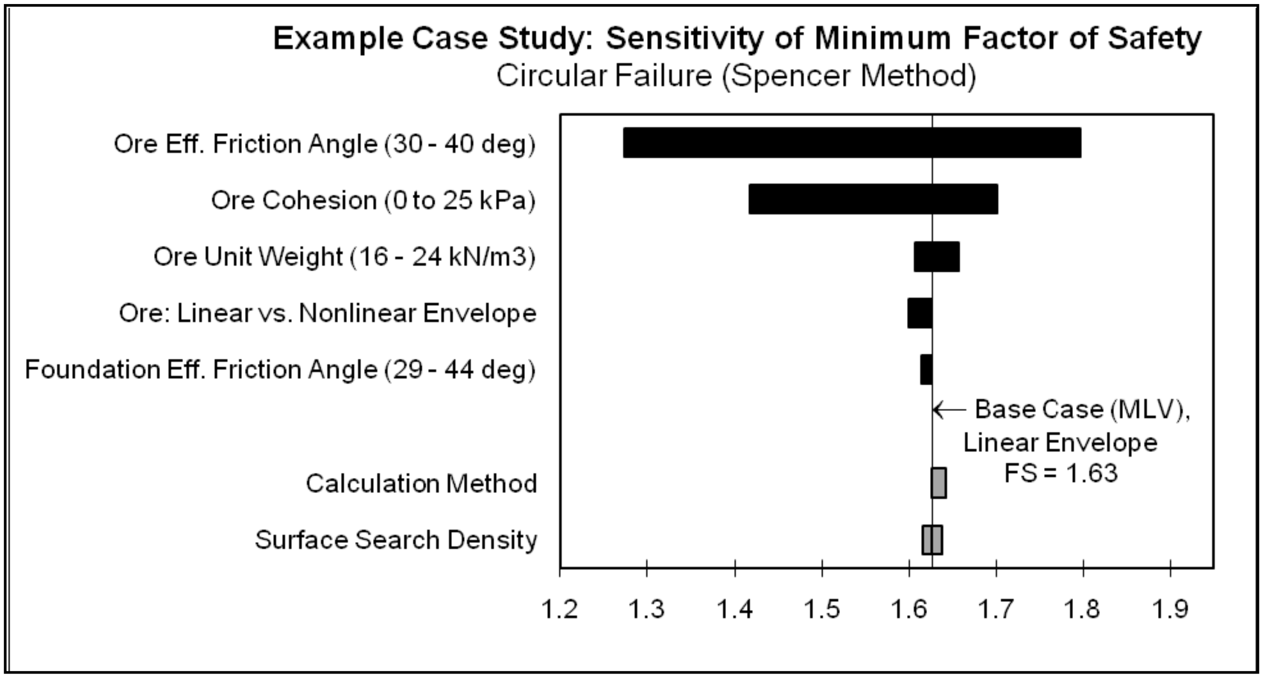


Figure 10. Sensitivity of Analysis for Circular Failure



Liner Interface Strength Parameters: The effect of liner interface friction angle on block failure is similar to that of ore material friction angle, but the sensitivity to liner interface cohesion is more pronounced than that for ore cohesion value. For liner interface friction angle and cohesion ranges of 17° to 27° and 0 kPa to 60kPa, the factor of safety varied from approximately 1.32 to 1.61 and 1.37 to 1.50, respectively. Understandably, the variation in liner interface shear strength parameters has a negligible effect on stability against circular failure passing through the ore material.

Liner Interface Strength Envelope Shape: The minimum factor of safety for failure through the liner was only slightly lower for the nonlinear envelope as compared to linear approximation. For example, using the Spencer method, the minimum factor of safety was 1.48 for properties based on the linear strength envelope, and 1.42 for properties based on the nonlinear strength envelope, (representing a 4 % difference). While the difference for this particular example is minor, it may be more significant for other projects and different shear test data. The difference in factor of safety for the nonlinear strength envelope and its linear approximation was primarily due to the difference in cohesion values used in the two cases.

Analysis Method and Algorithms: The Spencer, Bishop simplified and Janbu corrected methods generated very similar (e.g. within 5 % difference max) results for all of the stability analyses performed for the example case study (totaling over 100 runs). The average difference between the results from the three algorithms was only 2%. For the example case using MLV parameters, the analyses were performed for varying mesh densities and slope surfaces for the failure search method. As shown in Figures 9 and 10, the failure search density had a similar affect on the results as the use of different algorithms.

6.0 PROBABILITY OF FAILURE

The sensitivity of stability analyses of heap leach pads to design decisions and parameter values have been individually discussed in the previous sections. To provide an understanding of the overall range in possible analysis results and the compounded effects of multiple uncertainties, probabilistic evaluations were completed using both a spreadsheet method based on the "three-sigma-rule" and probabilistic tools available in Slide 5.0 (Rocscience). Probabilistic analysis provides a sense of the statistically expected factor of safety and the probability of failure.

The MLVs for each parameter were used to calculate the "most likely factor of safety", or FS_{MLV}, which is represented by the base case line in Figures 9 and 10. These FS_{MLV} values are typically the final results for stability analyses when a probabilistic evaluation is not included. The probabilistic study was conducted utilizing the previously discussed concepts of LCV and HCV for the analyses. The purpose of considering the LCV and HCV when performing the analysis is to obtain a feel for the reliability of the design conditions, with a larger range representing higher uncertainty. For the example study, since test data for the materials is limited or not available, consideration of the entire range of potential property values was imperative.

To complete the probabilistic analysis, a statistical distribution shape must be assumed for the values for each parameter. Selection of the distribution shape should be based on the level of knowledge available for each parameter. If only minimum (LCV) and maximum (HCV) values are confidently known for the parameter, then a uniform distribution should be used, with equal probability that any value between the LCV and HCV may occur. If the MLV is known as well, then a triangular distribution would be appropriate.

For probability analyses based on normal distributions, the standard deviations for each parameter must be identified. Since the standard deviations were not known for the parameters in the example study, the "Three-Sigma Rule" was applied to estimate the standard deviation, σ, for each parameter as follows (Duncan 2000; Thiel 2008):

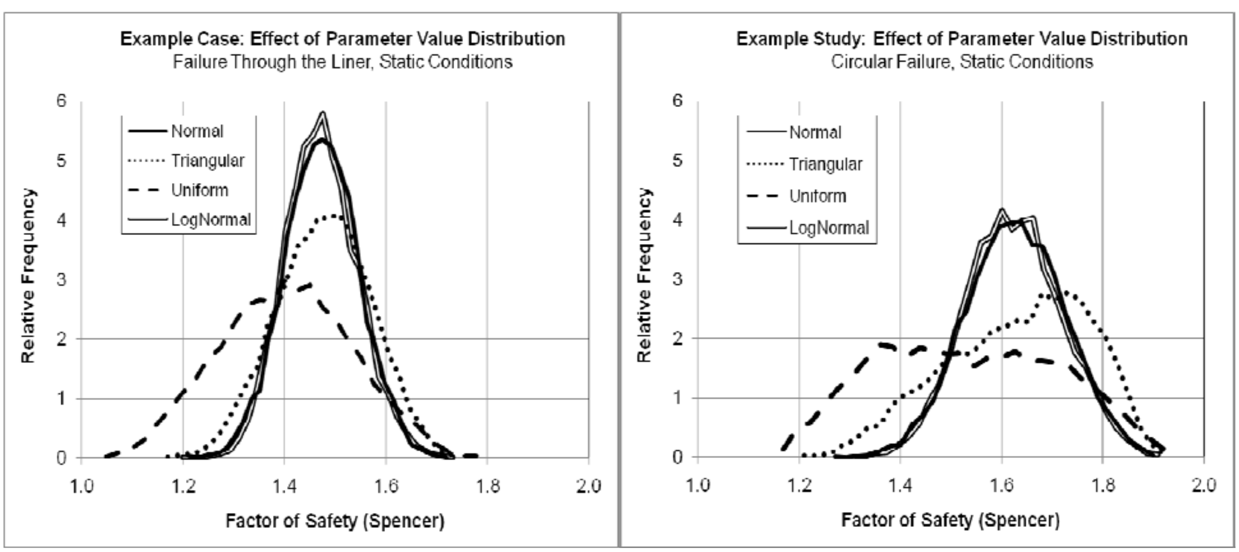
$$\text{standard deviation} = \sigma = \frac{\text{HCV} - \text{LCV}}{6}$$

This Three-Sigma-Rule is based on the premise that 99.7% of the values for a normally-distributed parameter will fall within three standard deviations of the mean value. However, using this rule for our analysis suggests that we know more about the expected parameter values than just the mean, minimum and maximum values.

There is typically not enough test data available to confidently select a particular distribution type beyond uniform or triangular. Even when extensive geotechnical testing has been performed, often no one distribution is conclusively the best fit for the data. For example, Baecher et al. (1983) analyzed various tailings properties from over 40 mine sites, and often found that even for simple properties such as unit weight, multiple distributions could be applied to the data and achieve a 5% goodness-of-fit.

Based on these considerations, Monte Carlo probabilistic analyses were performed in Slide 5.0 to evaluate the effect of the assumed distributions for each parameter being studied. The Monte Carlo analyses were based on 10,000 samples with parameter values chosen from the defined ranges (i.e. LCV, HCV) and statistical distributions. The results are shown in Figure 11. For the example case presented, all of the parameters were assigned the same distribution shape for simplicity. However, when enough data is available, each parameter should be evaluated individually, to determine the appropriate distribution for each.

Figure 11. Results of Probability Analyses



For this particular example study, the use of normal, lognormal, beta (not shown), or gamma (not shown) distributions for all of the parameters produced similar results, as shown in Figure 11. Other designs may be more sensitive to the chosen distribution types though, and the effect should be evaluated on a case-by-case basis. The wide range of the factor of safety results for the uniform distribution represents decreased design reliability with increased uncertainty in parameter values. Also, despite the inclusion of multiple sources of potential variability, the mean factors of safety from the probabilistic analyses were very similar to the deterministic results, which were based solely on the "expected" (MLV) values. This is not always the case, particularly for highly sensitive systems, and the use of probabilistic analyses in addition to deterministic methods is highly recommended.

7.0 CONCLUSIONS

An example case study of a stability analysis of a heap leach pad and all the potential sources of variability were presented. In addition, a collection of data from several heap leach projects was provided for various geotechnical properties.

For the example case presented, despite the range of values used for each parameter, the results of the probabilistic analyses were similar to the simple deterministic factor-of-safety analysis. While this may be true for many other designs, it may not be for many others. Gold, silver and copper heap leach pads are the highest lined-fill structures in the world (Breitenbach 1998) and careful consideration must be given to the actual reliability of the stability design. In short, in spite of significant laboratory testing and field observation, engineering judgment is required to select an appropriate stability analysis method (deterministic or probabilistic) on a case-by-case basis.

Unfortunately, there is no rational procedure and background for determining the factor-of-safety margins and reliability required (Thiel and Smith, 2003). Even if extensive test data is available, the actual values chosen for the material properties are still a decision made by the designer, and not a direct representation of the actual conditions. These are the issues of further reflection.

There are additional sources of variability that were not evaluated in this paper, such as variations in the assumed phreatic surface above the liner and permeability of the ore, which should also be considered in heap leach project designs.

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