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Quantile Regression as a Tool for Investigating Local and Global Ice Pressures

Paul Spencer and Tom Morrison, Ausenco, Calgary, Alberta, CANADA

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Abstract

In the companion ATC 2014 paper by Morrison and Spencer, the quantile regression data processing technique was introduced. In the current paper, we use the technique to estimate the local pressure trend at various quantiles, namely 90%, 99% and 99.9% and to revise the recommendations in ISO 19906 (ISO International Standard. Petroleum and natural gas industries —Arctic offshore structures). In addition we make recommendations on the form and parameters of the random pressure distribution for use in Monte-Carlo methods. The quantile regression method is also used to analyse global pressure data for wide structures for both the Baltic ice measurements and the Arctic ice measurements and compare with the expressions provided in ISO. The alternative statistical analysis provided in this paper indicates that changes to the ISO formulations are required.

Introduction

In the companion paper (Morrison and Spencer, 2014) the quantile regression method of analysing data was introduced. The example data set was the local pressure for ice crushing failure (Masterson et al. 2007), who used least-squares regression along with binning of the data in deriving the design pressure guidelines. This result has been incorporated into an offshore structure standard (ISO, 2010). Using this data set it was shown (Morrison and Spencer, 2014) that a fuller picture of the data could be obtained using a method that does not require binning or grouping of the data. As was also demonstrated, the power-law fit parameters determined using the quantile regression are different from the recommendations contained within ISO (2010).

In this paper we use the quantile regression approach to estimate what would be an appropriate probability distribution function for the ice crushing pressure at local scale and to produce new recommendations for local design pressure at a range of quantile, or equivalently a range of probabilities of exceedence. In addition we use the quantile regression method to analyse the data used in generating the Arctic global and the Baltic global pressure recommendations given in ISO (2010) and to compare with the ISO recommendations.

Local Pressure

The quantile regression analysis of the local pressure data (Masterson et al. 2007) for quantiles between 0.05 and 0.95 are shown in Figure 1 where the pressure data were fitted to a power law on area. The plot also includes 351 pressure-area data points used in the analysis. As may be seen from Figure 1 and from Morrison and Spencer (2014), the slopes of the various quantile lines vary with the value of the quantile.

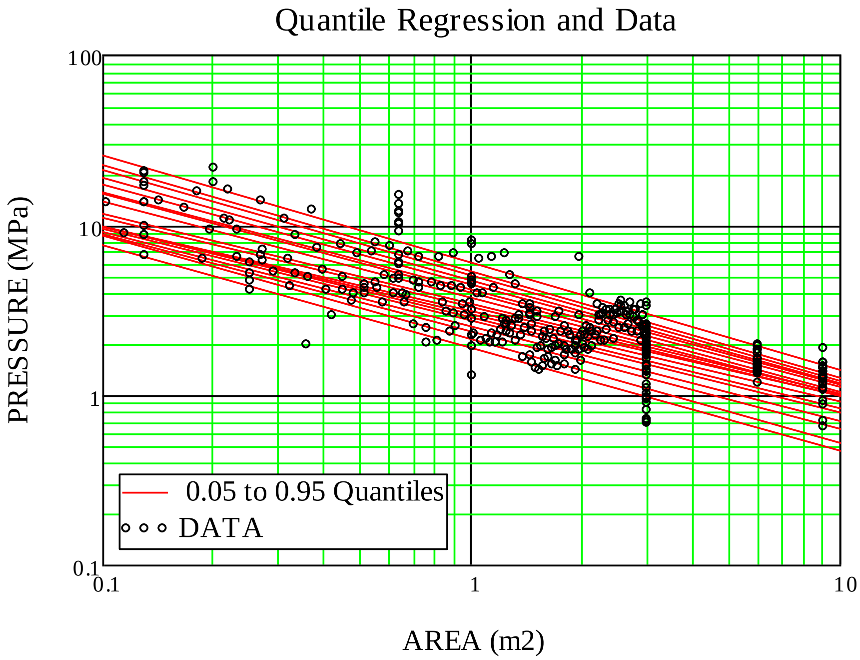

Figure 1 : Quantile Regression to a power law on area

The assumed fitting function, a power law on area, can only be approximately correct over a limited range of area. At small contact area, the pressure tends to infinity and at large contact area the pressure tends to zero. Both of these limits are physically unreasonable. In ISO equation A.8-65 (ISO, 2010) the guideline for local pressure clamps the pressure at 1.48 MPa for area between 10 m^2 and 40 m^2 recognising the limitations of the power law function at large area. Furthermore in the CSA recommendations (CSA, 2004), the local pressures are provided in a probabilistic form along with a deterministic form. Inspection of the probabilistic form indicated that the probabilistic 0.99 quantile pressure matched the CSA deterministic form at area of less than 1.0 m^2, but at larger area the probabilistic form significantly exceeds the deterministic pressure. Again this CSA formulation recognises that the power law on area has limitations at the larger contact area.

For the reasons stated above, the fitting function in the quantile regression was generalised to include an additional variable, a pressure offset that addresses the large area limit. Performing a quantile regression with three variables, i.e. multiplier, exponent and offset, it was found that over a wide range of quantiles, the value of the offset was approximately 0.77 MPa. The value of the offset was generally more than 2 standard errors away from zero indicating a valid trend and that the offset in not likely to be zero as had been previously assumed. A second quantile regression was then performed using a fixed offset determined from the mean value of the offset from the first regression. This second regression then gave a low variability value of the pressure-area slope (exponent) over a wide range of quantiles, in contrast to the regression lines shown in Figure 1. A third quantile regression was then done using a fixed value for the slope using the mean value of the slope from the second regression and the fixed value of the offset. The output of this third quantile regression is then the value of the multiplier of the power law term. The results of these three successive quantile regressions are provided in Table 1. From Table 1 it can be seen that the standard error in the multiplier is approximately 5% over most of the quantile range, increasing at the extreme low or high quantiles.

For comparison with the quantile regressions performed, a “standard” non-linear least-squares regression to a power law plus an offset was done using the same 351 data points. From this analysis we obtained Multiplier = 4.184 ± 0.672 (MPa), Exponent = -0.558 ± 0.065 and Offset = -0.258 ± 0.591 (MPa). On the basis of the least-squares regression, the standard error on the offset is large and there is not any evidence that the offset is different from zero. This comparison emphasises that exploring the local pressure data using quantile regression provides much more information than is obtainable from a least-squares regression to the mean.

The third regression quantiles given in Table 1 are also shown in Figure 2. For presentation purposes not all of the quantiles have been shown in Figure 2. Also shown are the 351 input data points and the ISO (2010) local pressure line. The plot has been extended to an area of 40 m^2 because ISO indicates that the local pressure is valid to this area. As discussed in Morrison and Spencer (2014), at small area the ISO line is lower than the 0.99 quantile and in this case corresponds to approximately

0.98 quantile whereas at 10 m^2 the ISO line corresponds to a 0.80 quantile. At an area of 10 m^2 the ISO line would then be expected to have a 20% chance of being exceeded.

On the basis of the analysis presented here, we recommend that the functional form for the local pressure-area relationship includes a non-zero offset and a fixed value of the exponent (slope).

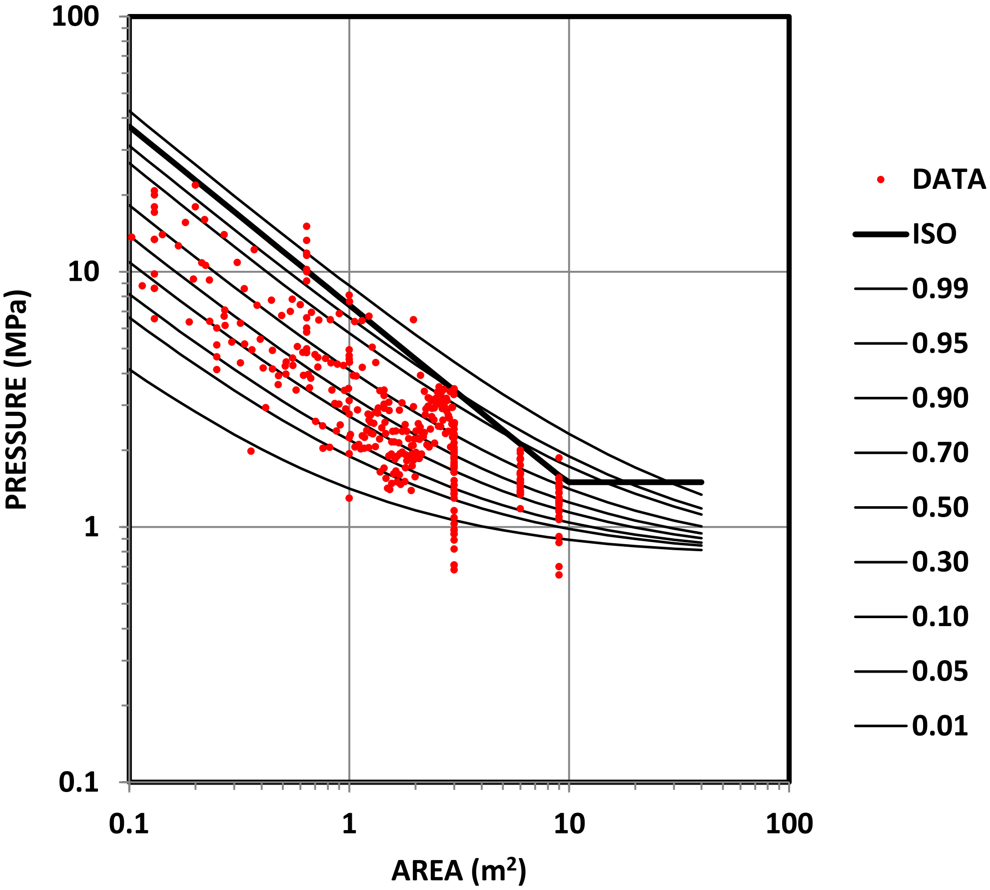

Figure 2 : Quantile Regressions Fitted to: Pressure = Multiplier x Area$^{-0.716}$ + 0 766 MPa; Local Pressure

For use in a probabilistic analyses of ice forces and to provide estimates for quantiles at higher values e.g. 0.999, the type of the probability distribution at a particular area and the parameters describing it are required. Since the form of the probability distribution is not known, the Gumbel, Lognormal and Gamma distributions (Wikipedia, 2013) were evaluated as possible candidates. Each of these distributions can be specified by two parameters, namely the mean and the standard deviation. The mean and standard deviation can be determined from just two independent quantile lines (Cook, 2010), but which two should be used? We used a range of the quantiles and conducted a least-squares analysis to determine the best-fit mean and best-fit standard deviation of each candidate distribution at a particular area. The best fit values were determined from a grid-search optimisation method over a range of mean values and standard deviation values and retaining the combination that gave the lowest value of the mean square of the deviations. This was done at areas of 0.1, 0.2, 0.5, 1.0, 2.0, 5.0 and 10.0 m^2 for each of the three probability distributions. In general the the best fit mean and standard deviation were different for the Gumbel, Lognormal and Gamma distributions. The sets of the mean and the standard deviations were then fitted to a power law on area using a non-linear least-squares regression. The mean and standard deviations of the three candidate distributions at an area of 1.0 m^2 were then used to determine the pressure that corresponded to a particular quantile and compared with the results of the quantile analysis at the same area.

Table 1 : Quantile Regression Results on Local Pressure Data

Quantile	Offsett from 1st regression (MPa)	Exponent from 2nd regression	Multiplier from 3rd regression (MPa)
0.01	n/a	n/a	0.065 ±0.146
0.05	n/a	-0.879 ±0.069	1.126 ±0.077
0.10	0.263 ±0.32	-0.770 ±0.067	1.426 ±0.070
0.20	0.722 ±0.29	-0.728 ±0.052	1.691 ±0.077
0.30	0.737 ±0.25	-0.676 ±0.053	1.952 ±0.093
0.40	0.792 ±0.28	-0.684 ±0.060	2.247 ±0.118
0.50	0.895 ±0.26	-0.722 ±0.050	2.514 ±0.131
0.60	0.682 ±0.29	-0.722 ±0.045	2.942 ±0.170
0.70	0.639 ±0.36	-0.714 ±0.040	3.358 ±0.180
0.80	0.895 ±0.39	-0.729 ±0.044	4.114 ±0.232
0.90	0.410 ±0.55	-0.713 ±0.041	4.979 ±0.307
0.95	n/a	-0.699 ±0.087	5.850 ±0.479
0.99	n/a	-0.700 ±0.158	8.053 ±1.003
Mean	0.766 ±.038	-0.716 ±0.027	

The analysis indicated that using quantiles ranging from 0.05 to 0.95, the Lognormal function provided for better overall matching than either the Gumbel or the Gamma distributions. The 0.99 quantile pressure was not used in the matching since it has a larger error than the lower value quantiles, as may be seen from the data presented in Table 1. However, for Monte-Carlo methods or for providing deterministic values of the design pressure, the higher value quantiles are of more interest than quantiles at, for example 0.1. Using the higher value quantiles ranging from 0.6 to 0.95 it was found that the Gamma function provided for a better fit at quantiles larger than 0.5 and at lower quantiles the Lognormal function provided for the better fit. These two distribution functions when fitted to the quantiles are illustrated in Figure 3. The part of the graph between quantile 0.9 and 1.0 is also shown with an expanded scale. We recommend that the Gamma distribution be used for all values of quantiles and the parameters defining the mean and standard deviation are given in Table 2.

Table 3 provides the deterministic pressure guidelines for quantiles between 0.90 and 0.999 calculated using the calibrated Gamma distribution function given in Table 2. The reason for providing values at a range of quantiles rather than at just 0.99 is that the ISO guidelines (Section A.8.2.5.1) indicate that the deterministic pressure corresponds to a 100 year return period. The probability of exceedence required to match the 100 year return period would then depend on the annual number of interactions. Design pressure values at other quantiles can be calculated using the information provided in Table 2. Note however, since extrapolation is being used to determine the higher value quantiles, that caution be used if quantiles larger than 0.999 are calculated.

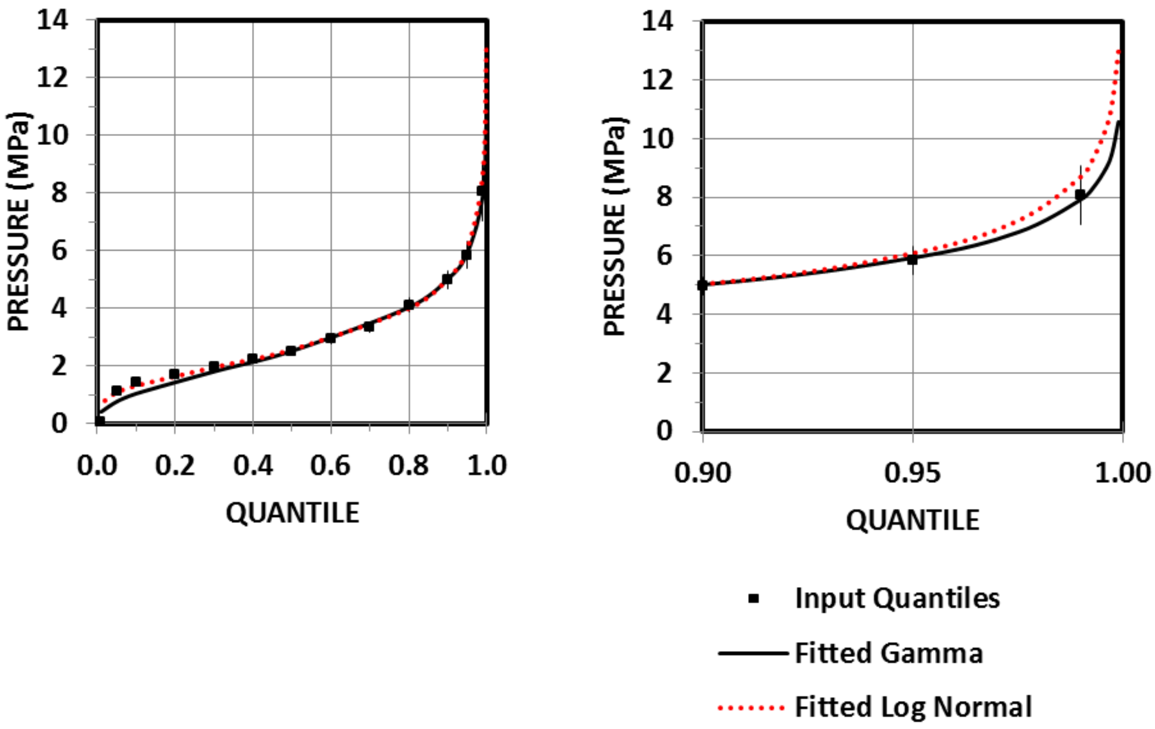

Figure 3 : Gamma and Log-Normal Distributions Fitted to Quantiles; Local Pressure Data

The design pressures from Table 3 are shown in Figure 4 along with an extension to 40 m^2, along with the 351 data points used in the generation of the curves and the current ISO local pressure guideline. Note that the 0.99 quantile, i.e. 1% probability of exceedence line, is the second line, the uppermost line corresponds to the 0.999 quantile. The small area parts of these curves are also shown on a linear scale in Figure 4. The values provided in Tables 2 and 3 were calibrated using pressure data for area between 0.1 and 9.0 m^2, however we suggest that the range of applicability can be extended to an area of 40 m^2. The new recommendations do however result in design pressures larger than the current ISO guidelines. This is due to two main factors: the first is that the current ISO guidelines do not correspond to a 1% probability of exceedence value; the second is from the observation that a pressure offset exists in the data set thereby increasing pressures at the larger area. From Figure 4 we can see that at an area of 10 m^2 the new design pressure is 2.4 MPa compared with 1.48 MPa for the current guideline. Note that the data at 3.0, 6.0 and 9.0 m^2 were from readings collected on the Molikpaq structure in the Canadian Beaufort Sea (Jefferies and Spencer, 1989). The frequency response of the Medof Panels used to collect the data is low (Spencer, 2013) and thus the peak amplitude of shorter duration events may be under-represented. In addition there are only 15 data points at each of 6.0 and 9.0 m^2 areas and the highest value data point would then represent an approximately 0.93 quantile not a 0.99 quantile.

Table 2 : Parameters defining Gamma distribution function fitted to Power law on Area

Power Law Fit (1)	Mean of Gamma	Stdev of Gamma
Multiplier (MPa)	2.832	1.630
Exponent	-0.715	-0.716

(1) Add 0.766 MPa offset to values calculated when using Table 2

Table 3 : Design Local Pressure at Various Quantiles

Quantile	Local Design Pressure Relationship (Valid 0.1 to 40 m^2)
0.999	Pressure (MPa) = 10.569 * $Area^{-0.716}$ + 0.766 (Area in m^2)
0.990	Pressure (MPa) = 7.916 * $Area^{-0.716}$ + 0.766 (Area in m^2)
0.950	Pressure (MPa) = 5.933 * $Area^{-0.716}$ + 0.766 (Area in m^2)
0.900	Pressure (MPa) = 5.018 * $Area^{-0.716}$ + 0.766 (Area in m^2)

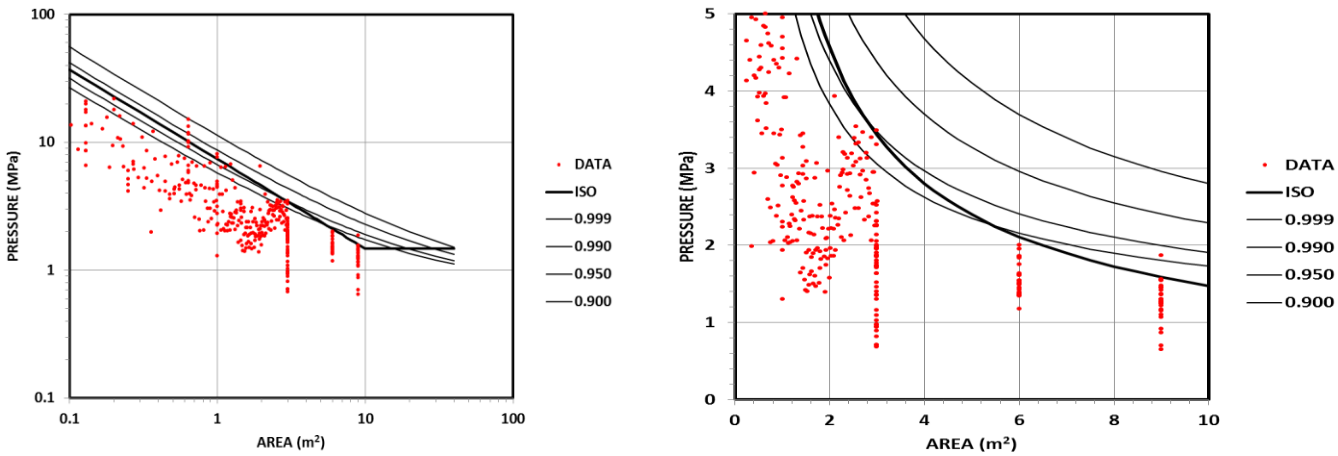

Figure 4 : Local Pressure Design Values at Various Quantiles (Log Plot - left and Linear Plot - right)

Global Pressure – Arctic Design

The global pressure recommendations for Arctic structure design are provided in ISO, Equation A.8-21 (ISO, 2010) when the ice thickness is larger than 1.5 m, the aspect ratio is larger than 2.0 and crushing is the mode of ice failure. The deterministic design curve is given in Equation 1 as a product of power laws on ice thickness and aspect ratio. The multiplier Cr has a value of 2.8 MPa and is indicated in ISO to correspond to ELIE loading (100 year return period).

$$\text{Pressure (MPa)} = Cr\,(h/h_0)^{-0.3} * (w/h)^{-0.16} \qquad \text{Equation (1)}$$

Cr = 2.8 MPa; h = Ice Thickness (m); h_0 = 1.0 m; w = Interaction Width (m)

The data used in the generation of Equation 1 was mainly from the Molikpaq when located in the Canadian Beaufort Sea (Kärnä and Masterson, 2011). These authors indicate that the ISO curve is an envelope to the observed data points except for a small number of observations. In other words, an informal quantile approach was being used.

The data presented by Kärnä and Masterson (2011) was extracted from Timco and Johnson (2004). For completeness we present the load data that will be used for the current quantile regression analysis in Table 4. From Table 4 it can be noted that the ice thickness varied from 0.7 m to 7.0 m and the interaction width varied from 60 m to 105 m. Note that only seven of the data points are for ice thickness greater than 1.5 m, the lower limit for use of equation 1. The interaction pressure is calculated from the interaction load divided by the contact area (Thickness * Width).

Table 4 : Arctic Global Load Data from Molikpaq

60 m Width		75 m Width		90 m Width		95 m Width		105 m Width	
Thick (m)	Load (MN)	Thick (m)	Load (MN)	Thick (m)	Load (MN)	Thick (m)	Load (MN)	Thick (m)	Load (MN)
0.9	44	0.9	64	0.8	84	0.7	60	0.8	79
0.9	70	0.9	60			0.7	63	0.8	93
1.2	77	0.9	59			0.7	66	0.8	83
0.7	69	0.9	50			0.7	60	0.8	104
0.7	78	0.9	58			0.7	65		
0.7	53	0.9	50			0.7	77		
0.7	48	1.2	99			0.8	87		
0.7	57	0.9	73			0.8	85		
0.6	34	0.9	71						
0.6	48	0.9	77						
3.0	210	0.8	58						
3.0	215	0.7	61						
7.7	466	0.7	54						
7.0	361	0.8	61						
		1.2	110						
		0.6	67						
		3.0	181						
		2.7	203						
		2.7	208						

Quantile analysis was performed on the interaction pressure (calculated from Table 4) with the multiplier and the two exponents in Equation 1 as fitted variables. The analysis indicated that the value of the multiplier was not well defined, for example at the 0.935 quantile, the multiplier was 30.6 ± 34.9 MPa. However, by treating the multiplier as a fixed value rather than as a variable, the quantile regression on the two exponents resulted in exponents similar to that given in Equation 1. These results are illustrated in Figure 5 where the standard errors for the two fitted exponents are provided for the 2.8 MPa line. The error bars for the 2.4 MPa and 3.2 MPa lines are similar but are not shown for clarity.

In Figure 5, the ISO values given in Equation 1 are indicated by the red dot and correspond to a quantile of 0.925. However, other combinations of multiplier and exponents also fit the data at the same quantile as shown by the black and green dots. Thus the analysis has indicated that the parameter values given in Equation 1 are not unique. In addition, the ISO values given in Equation 1 correspond to a quantile of 0.925, not the implied 0.99 quantile. From a practical perspective, these alternative combinations of multiplier and two exponents provide pressure values within 5% of that given by Equation 1 for ice thickness between 1.5 and 6.0 m and for widths between 25 and 200 m.

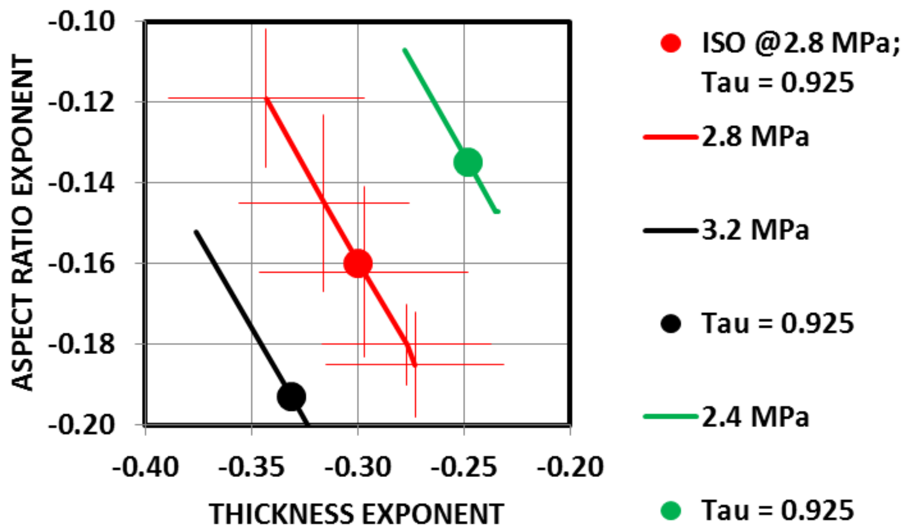

Figure 5 : Quantile Analysis to Arctic Global Data (Table 4)

Additional information on the value of the multiplier was obtained from a quantile analysis of data (25 data points) that were from a single column of Medof Panels measurements (Jefferies and Spencer, 1989, see Table 8.2; Kärnä and Masterson, 2011). The data points are shown in Figure 6 along with examples of quantile regressions to a power law on thickness. The ice thickness varied from 0.6 to 2.7 m. Figure 7 shows the values of the multiplier and the thickness exponent for a range of quantiles. In Figure 7 is also shown a fitted Gamma distribution function to the multiplier using quantile values of 0.2 or greater. Due to the small number of input pressure values, the multiplier trend with quantile is not smooth. However, the Gamma fit is judged to be a reasonable representation. For the purpose of extrapolation to the 0.99 quantile, caution should be used in the predicted value of 3.6 MPa. From the quantile regression, at a multiplier of 2.8 MPa, i.e. the ISO value, the quantile is 0.875. From the analysis using the data shown in Table 4, the ISO values corresponded to a quantile of 0.925. Combining these two estimates indicates that the ISO Equation (Equation 1 above) corresponds to a quantile of 0.90, not a 0.99 quantile.

Shown in Figure 7 are also the values of the thickness exponent. As can be seen the uncertainty in the value is large. However, the mean value using the data presented in Figure 7 is -0.248 with a scatter of 0.097. Treating the data points as independent, resulted in a standard error of the mean of 0.023. The thickness exponent value from the present analysis is similar to but less strong than the -0.30 value used in ISO.

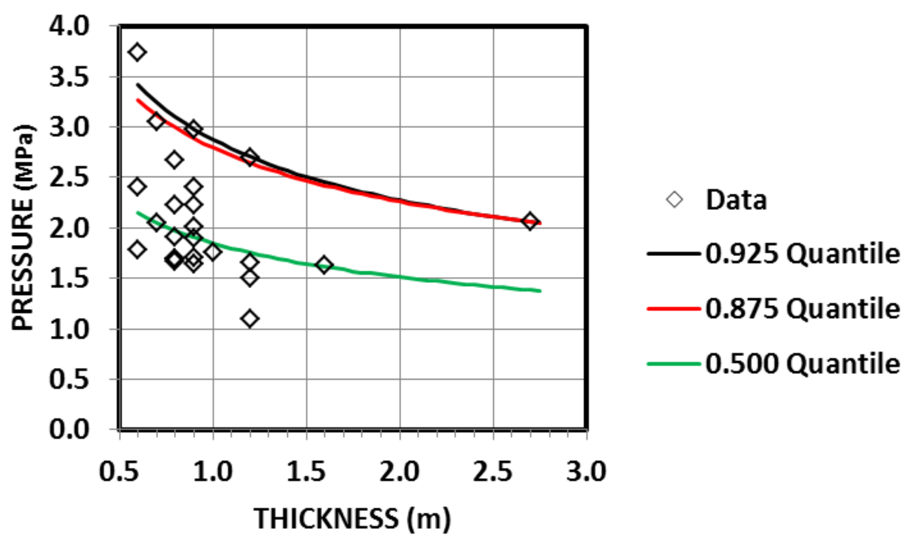

Figure 6 : Single Medof Panel Column Data and Quantile Regression

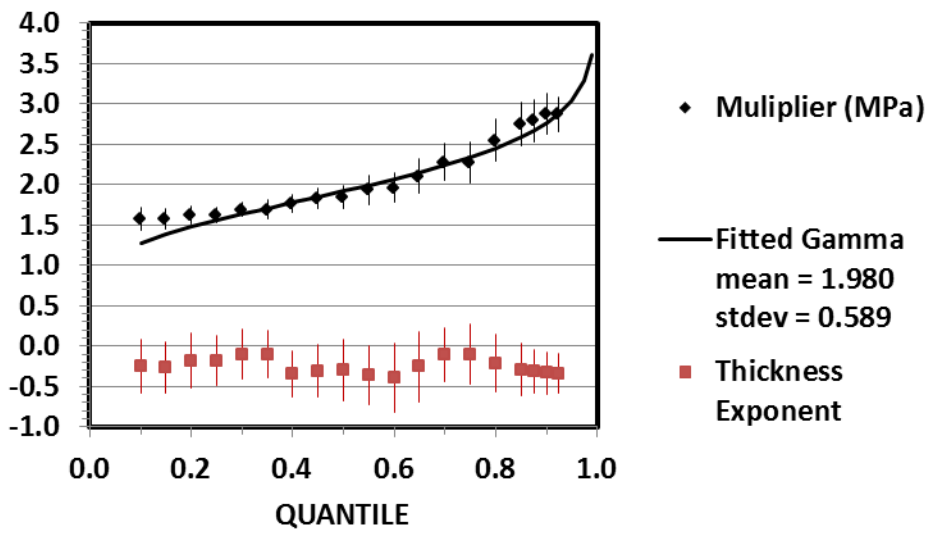

Figure 7 : Multiplier, Fitted Gamma and Thickness Exponent as function of Quantile - Single Medof Column

Table 5 : Expected Annual Maximum Pressure at 1.0m^2 - Global Arctic

Events/Year	1	2	5	10	20	50	100	200	500
Expected Annual Max Pressure (MPa)	1.98	2.32	2.70	2.97	3.23	3.54	3.77	4.00	4.26

Using a random Gamma distribution with parameters described in Figure 7 the expected annual maximum pressure at 1.0 m^2 can be calculated for a range of events per year. The results of these calculations are given in Table 5. One can see from interpolation of Table 5 that the current guideline Cr = 2.8 MPa corresponds to an expected annual maximum of 6 events per year. Alternatively the annual expected pressure with 200 events per year is equivalent to the 100 year expected pressure with 2 events per year. From Hardy et al., (1996) describing the Amualigak I-65 deployment during 1985-1986:

> *The structure experienced a number of multi-year ice interactions in mid winter. The multi-year ice was representative of extreme winter ice conditions but the floes that impacted the Molikpaq were not particularly severe.*

In summary, we have found that the ISO global pressure guideline given in Equation 1 corresponds to a 0.90 quantile. The multiplier that corresponds to a 0.99 quantile is 3.6 MPa, approximately 30% larger than the ISO value. The analysis suggests that the ISO Cr value of 2.8 MPa is unconservative. The values of the ISO exponents on thickness and aspect ratio are shown to be reasonable but not unique. The quantile analysis presented provides a direct link between the data set used and the recommendations provided in ISO (2010). We also note that the majority of the pressure values used to support the ISO recommendation (see Table 4 and Figure 7) were from ice sheets with a thickness of less than 1.5 m, some as thin as 0.6 m. We suggest that the 1.5 m minimum thickness restriction contained in ISO is overly restrictive and can be relaxed.

Global Pressure – Baltic Design

In ISO (2010) there are recommendations on global pressures based on data collected on an instrumented lighthouse located in the Baltic Sea (Kärnä and Qu, 2006). We use some of this data to further illustrate the quantile regression method and to provide additional insights into the data. We use data that corresponds to brittle crushing at the higher drift velocity (pers. com. Yan Qu, 2013). The first example is for data on a single panel when the drift direction is approximately normal to the panel. The panel had an effective width of 0.97 m. In this data set are 50 data points where the ice was between 0.14 m and 1.13 m in thickness. The pressure-thickness input data along with some results of a quantile analysis to a power law on thickness are shown in Figure 8.

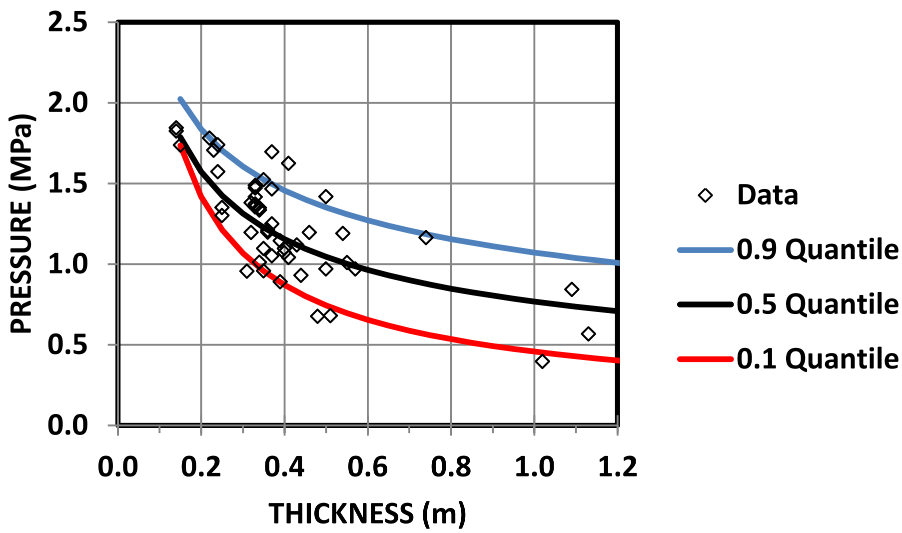

Figure 8 : Single Panel Data Baltic Lighthouse

The values of the multiplier and thickness exponent are shown in Figure 9 as a function of quantile. The standard errors from the quantile regressions are also shown. In general, the errors increase at large quantiles. Using a least-squares fit to the mean of the data, a value of exponent of -0.5 was found (pers. com. Yan Qu, 2013). From the quantile analysis presented, this value is reasonable for quantiles between 0.2 and 0.8 but from Figure 9 there is evidence that the exponent may vary with quantile. In other words the exponent may be different at the highest values of quantile than at the lowest values of quantile.

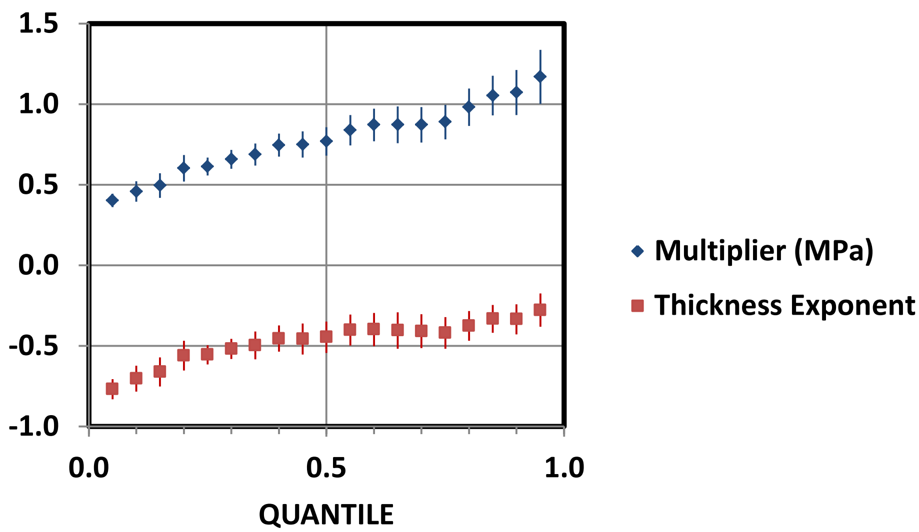

Figure 9 : Values of Fitted Parameters from Quantile Regression to Power Law on Thickness; Single Panel Baltic Data

The second example is again for brittle crushing but uses data from multiple panels so that the data is now a function of both thickness and width. The data set, comprising 507 readings (Kärnä and Qu, 2006), is shown in Figure 10 where the data has been grouped in various width ranges. Data from the winters of 2000, 2001, 2002 and 2003 are included. As may be noted from Figure 10, the pressure values for interactions with a width between 5 to 6 m are generally smaller than at smaller widths. In addition, the data suggests that the pressures are lower at the larger ice thickness.

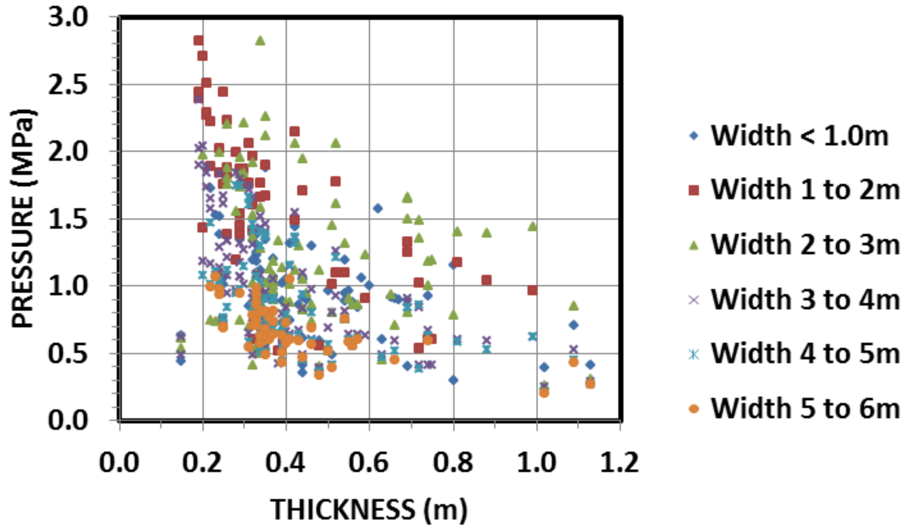

Figure 10 : Pressure Data from Multiple Panels for various interaction widths and thickness; Baltic Data

A quantile regression was done using a power law on actual thickness and on the actual width (the binning in Figure 10 was done for presentation purposes only). Three parameters were fitted, the multiplier and the values of the exponents on thickness and on width. These fitted parameters are shown in Figure 11 as a function of quantile. Treating the values of the two exponents as fixed values given by their mean value, another quantile regression was done providing values of the multiplier. The resulting multiplier values were then fitted to a Gamma, Lognormal or Gumbel distribution. The Gamma distribution was judged to provide the best fit and it shown in Figure 12 along with the results from the quantile regression. The information on the form and parameters for the pressure probability distribution is not easily obtainable from traditional analysis using least-squares to the mean of the data.

The results from the brittle crushing measurements can be transformed to a power law on thickness and aspect ratio as per Equation 1. In addition, the value of the multiplier from the Gamma distribution shown in Figure 12 at the 0.99 quantile (1% probability of exceedence), can be obtained and given by Equation 2.

Pressure (MPa) = Cr $(h/h_0)^{-0.48}$ $*(w/h)^{-0.24}$ Equation (2)

Cr = 1.23 MPa; h = Ice Thickness (m); h_0 = 1.0 m; w = Interaction Width (m)

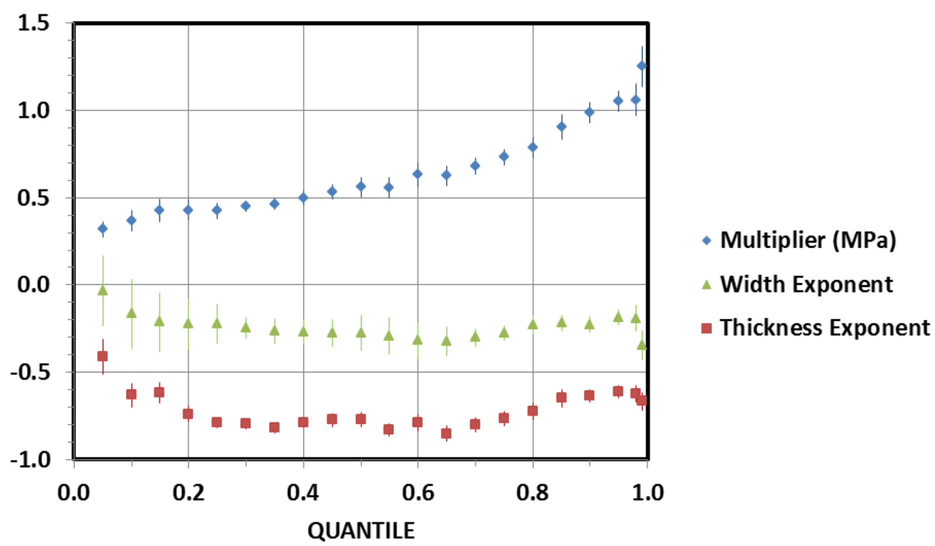

Figure 11 : Parameters from quantile regression on thickness and width; multiple panels Baltic data

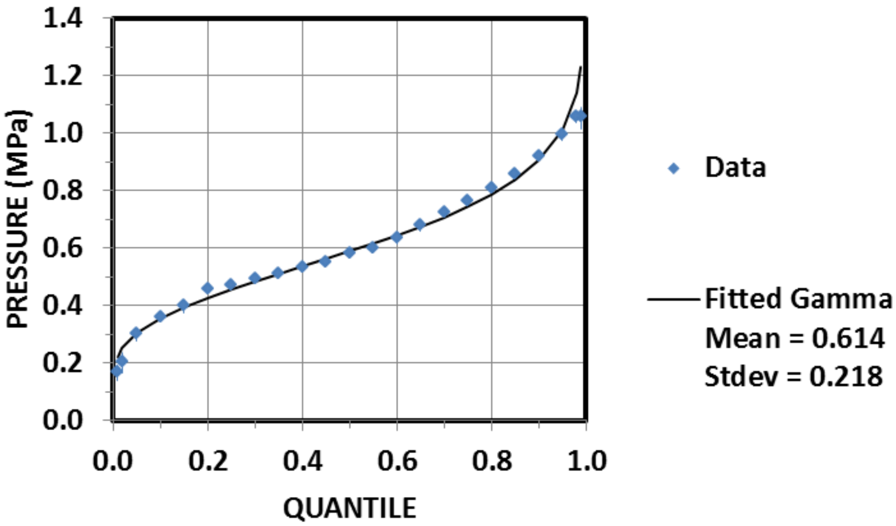

Figure 12 : Quantile Regression fitted to Pressure = Multiplier x Thick$^{-0.7174}$ Width$^{-0.2422}$

We compare the results of the quantile analysis with the Baltic wide structure recommendations in ISO (2010). For the quantile analysis we selected the multiplier that corresponds to the 1% exceedence as given in Equation 2. This comparison was done for ice thickness ranging from 0.1 to 1.5 m and for widths ranging from 0.75 to 7.0 m covering the observed range in the input data. The requirement that the aspect ratio was greater than 2.0 was also applied. The results of the comparison are shown in Figure 13 where the line represents the linear least-squares fit with intercept = 0.042 ± 0.027 and slope = 0.580 ± 0.015.

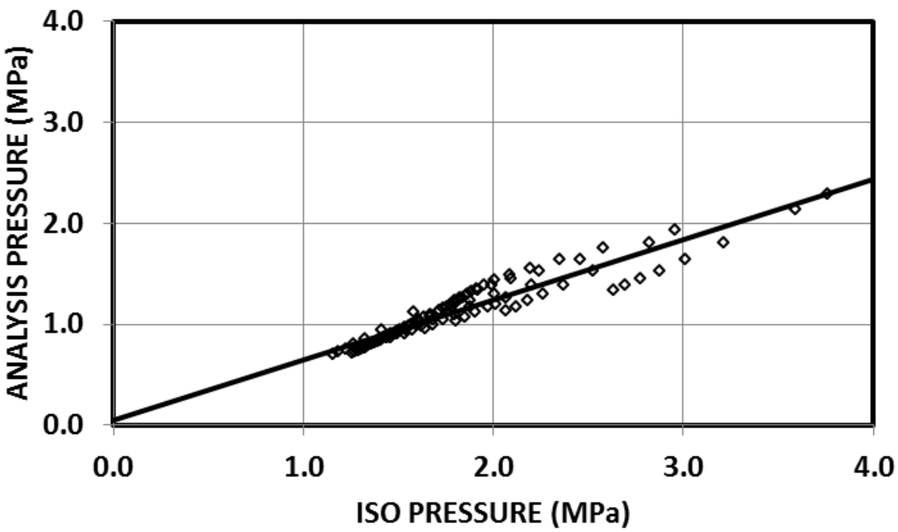

Figure 13 : Comparison between ISO and Quantile Analysis of the Baltic Data

From Figure 13 one can note that there is a good correlation (r^2 = 0.94) between the two formulations and that the fitted offset is small, indicating that the two different analysis methods are producing similar results even though different exponents are used in ISO and in the current analysis. The main difference between the current analysis and ISO is that the slope of the data in Figure 13 is not equal to unity. Kärnä and Masterson (2011) outline the derivation for the ISO Baltic recommendation and provide the following quote:

> *The value Cr = 1.8 MPa was derived by applying a multiplying factor of 1.4 on the values that yield global pressures at the level of expected annual maximum.*

The expected annual maximum value of the data illustrated in Figure 10 was calculated using 127 samples of a random Gamma distribution with parameters defined in Figure 12. The 127 samples were based on 4 years of data and a total of 507 data points. This calculation indicated that the annual maximum pressure is 1.3295 MPa. The annual maximum times 1.4 equals 1.861 MPa and is in good agreement with the quoted value of Cr = 1.8 MPa. The comparison indicates that the ISO line also corresponds to the 0.99 quantile value times 1.72. Lastly, the quantile analysis provides for a simpler connection between the input data and the recommendations contained within ISO.

Discussion and Conclusions

Data sets having a large scatter that varies with the measured parameters can be analysed using quantile regression. In this approach, the scatter is treated as the essential part of the data, not just a nuisance that has to be accommodated. More information on the data can be obtained using quantile regression than is obtained from least-squares analysis. The quantile regression analysis can determine, for example, pressure values at large values of quantiles and investigate if various exponents are constant. These derived values can then be used in the formulation of appropriate codes and guidelines. Furthermore, the technique can also be used to suggest appropriate probability distributions for the interaction pressure.

The comparison between the existing ISO guidelines and those of the current analysis are given in Table 6. In the ISO guidelines there are not any recommendations on probability distributions for the local or global cases.

Table 6 : Comparisons between ISO and New Analysis

Guideline Type	ISO (MPa)	New Analysis at 1% exceedence (MPa)	Gamma Probability Distribution Parameters at 1.0 m^2 (MPa)
Local	$P = 7.4\ Area^{-0.70}$	$P = 7.92\ Area^{-0.72} + 0.77$	$\mu = 2.832$; $\sigma = 1.630$ (+ 0.77)
Global Arctic	$P = 2.8\ h^{-0.3}\ (w/h)^{-0.16}$	$P = 3.6\ h^{-0.3}\ (w/h)^{-0.16}$	$\mu = 1.980$; $\sigma = 0.589$
Global Baltic	$P = 1.8\ h^{-0.5+h/5}\ (w/h)^{-0.16}$	$P = 1.23\ h^{-0.48}\ (w/h)^{-0.24}$	$\mu = 0.614$; $\sigma = 0.218$

For the local pressure, a new formulation for the deterministic pressure guideline (ISO, 2010) is proposed that is based on a power law plus a pressure offset. The values of the parameters given in Table 6 correspond to a 1% probability of exceedence. A calibrated Gamma pressure distribution function is also provided that may be used in Monte-Carlo analysis of interaction pressures.

The data supporting the Arctic global pressure trends in ISO were analysed and the quantile analysis results generally support the functional form within ISO. The analysis provided a direct link between the recommendations and the input data. However, we also indicate that the formulation within ISO is not unique and other combinations of multiplier (Cr), thickness and aspect ratio exponents can also match the input data. It was found that the ISO recommendations correspond to a quantile of 0.90 and that at 0.99 quantile the pressure multiplier (Cr) would be approximately 3.6 MPa in contrast to the 2.8 MPa value provided in ISO.

Pressure data from an instrumented lighthouse located in the Baltic Sea representing brittle crushing of ice were also analysed. Additional information on the values of thickness and width exponents were obtained along with estimates of the probability distribution function that describe the pressure data. The analysis provided similar pressures as in ISO but with different values of the thickness and the aspect ratio exponents. The pressure multiplier Cr = 1.8 MPa was verified in the current analysis as an ELIE level. The current analysis used a different approach from that resulting in the ISO recommendations.

The coefficients of variation for the three Gamma distributions given in Table 6 are 0.58, 0.30 and 0.36 respectively. This indicates that the relative scatter for the Global Arctic and Baltic are similar but smaller than for the local pressure.

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